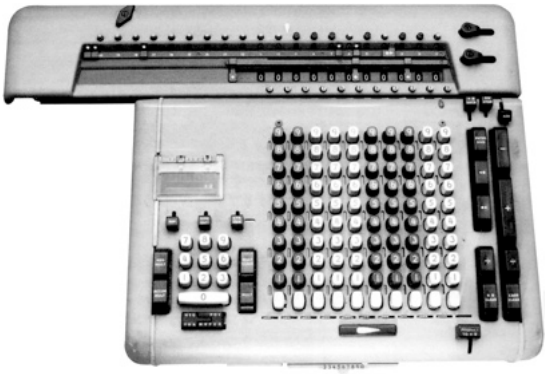
1

The aircraft design process

Recommended reading:

Nicolai & Carichner: Chapter 1
Brandt, Stiles, Bertin & Whitford: Chapter 1



2

Some key points for aircraft design

"Don't miss the forest for the trees" - here are some central issues it's very easy not to pick up on when you start out in aircraft design.

1. **Aim to make the aircraft efficient or effective as possible for its main task** (usually, what it will dissipate most energy doing – e.g. flying in cruise). This often requires paying attention to wing loading *W/S* (perhaps by reducing or increasing wing area *S*) as well as its (non-dimensional) shape to ensure that this is achieved (by getting to the right C_L point on the drag polar). As with all design tasks, aircraft design ends up being a compromise, (to meet other performance constraints) but remember to focus on the above principle (and figure out at the beginning what the "main task" really is).
2. The task of aircraft longitudinal centre of gravity (CG) placement is often misunderstood by students. The primary objective here is to get the CG (depends on aircraft mass distribution) the right amount forward of the aircraft neutral point (NP, depends on aircraft geometry) to achieve a suitable static stability margin (SM – usually of order 10%), and with the aircraft adequately controllable. This typically means that you may have to alter BOTH the mass distribution and geometry to achieve these goals. If you fix the geometry for efficiency and controllability, then you have to adjust the mass distribution to achieve a desired SM. (Naturally, the mass distribution will depend quite heavily on the geometry, but there are often component "point masses" that you can move around.) The last thing you want to do is to *add* weight to achieve what you need!

We spend a lot of time here addressing other topics but the two above are perhaps the most important to keep in mind when you're starting out.

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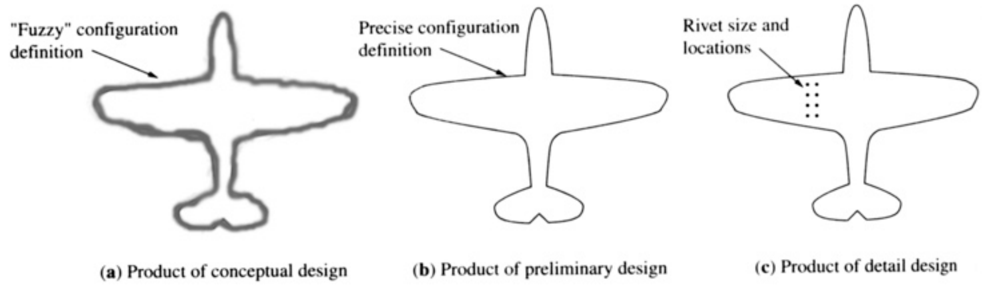
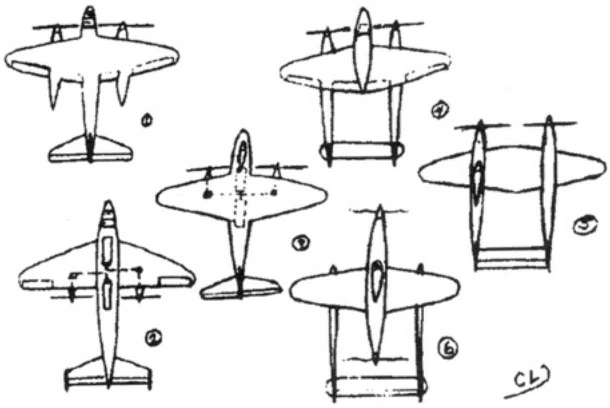
The staged approach to aircraft design

Design phases



Anderson

Lockheed design team alternative configuration sketches for the P38 Lightning, and the finished product.



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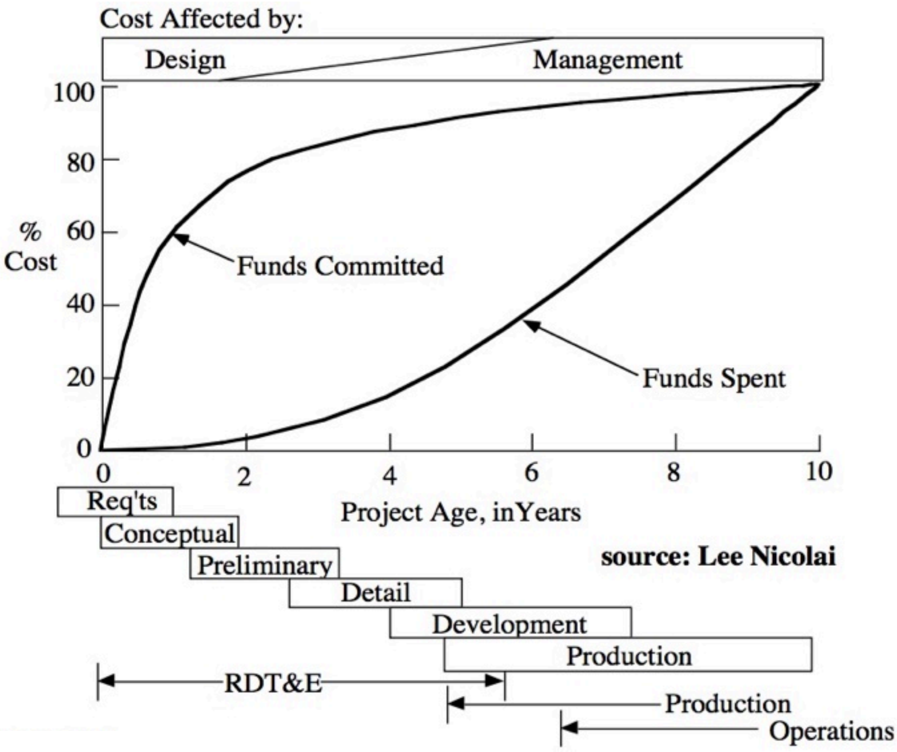
The importance of initial design phases

Decisions made early determine project cost



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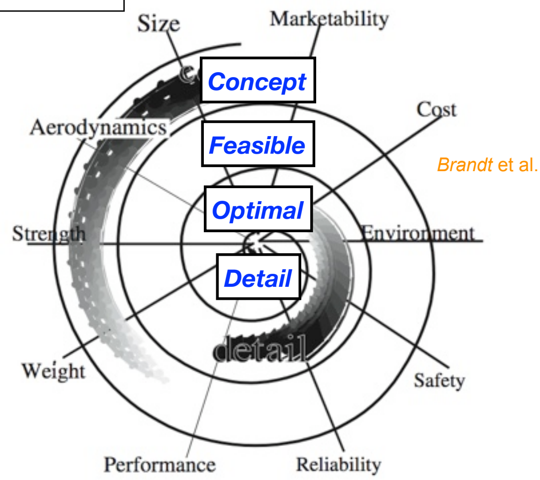
Aircraft design development concepts

Design iteration

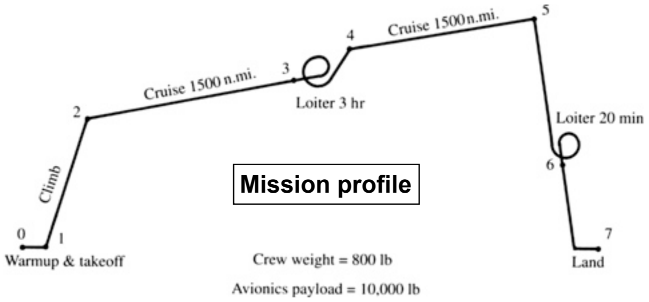


Initial design is typically an iterative process where the various design determinants are revisited a number of times and the design is refined:

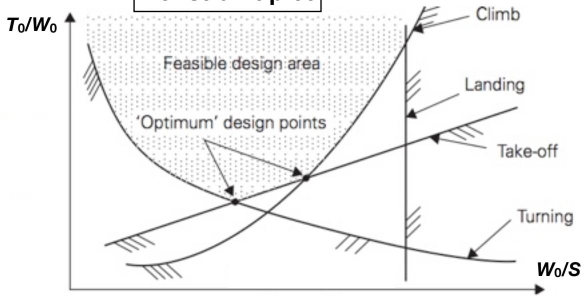
Conceptual → Workable → Optimal → Detail

The iterations typically require more and more work (and cost) in each phase.

Two common graphical presentations used in aircraft design are the <u>mission profile</u> which is used as an aid in maximum takeoff weight analysis W_0 and the <u>constraint plot</u> that shows combinations of the two remaining major design variables, thrust and wing area, T and S (as ratios to the maximum takeoff weight) which satisfy all the performance requirements.



Constraint plot



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Overview of aircraft initial sizing

1. Aircraft tend to be broadly similar within categories, which is helpful in the conceptual/initial design phase. Data correlations of previous designs are typically employed during initial design.
2. The specification or Request For Proposal (RFP) will typically provide a <u>mission profile</u> and
   a. <u>Performance requirements</u> e.g.
      i. Rate of climb capability at initial cruise height
      ii. Approach speed or landing field length
      iii. Takeoff field length
      iv. Maximum speed
      v. Rate of turn
   b. <u>Payload/range/endurance requirements</u>: energy/fuel use, e.g
      vi. Carry 150 passengers 1000km
      vii. Patrol for 24hrs

3. <u>The three primary aircraft design variables</u> are $\boldsymbol{W_0}$ (maximum take-off weight); $\boldsymbol{S}$ (wing area); $\boldsymbol{T_0}$ or $\boldsymbol{P_0}$ (engine max thrust/power at sea level) — all interdependent in the requirements. What to do?
4. Fortunately, *performance requirements* involve the ratios *W/S, T/W*, while *payload/range requirements* (which are basically energy requirements) are typically set up to include only aerodynamic and fuel consumption parameters (and often, speed). <u>To an extent, we can solve for the two different requirement types independently and iterate towards a converged solution.</u>
5. Once a <u>workable design</u> (i.e. one that satisfies all the criteria) is evolved, we can carry out <u>design trade</u> or optimisation studies to find better (e.g. lighter/less costly) solutions.
6. Also, we work to more tightly define the design, <u>make it flyable</u>, safer, easier to manufacture and maintain. After this point we start to enter the detail design phase.

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Preliminary design stages

1. **Conceptual Design** (speculative design) with General Arrangement and ballpark values of W_0, S, T_0
    a. General Arrangement sketch based on mission requirements, study of related aircraft
        i. Fuselage arrangement
        ii. Initial wing and tail planforms
        iii. Type of propulsion system and number/locations of engines
        iv. Undercarriage layout
    b. Initial maximum takeoff weight estimate (W_0)
        v. Make ballpark estimates of aerodynamic parameters for drag polar (C_{D0}, K)
        vi. Choose empty weight fraction correlation for aircraft category
        vii. Based on mission statement and 'Breguet' type range/endurance equations, estimate W_0
    c. Initial estimate of W_0/S (and hence wing area S) based on trend data for type
    d. Initial estimate of thrust:weight T_0/W_0 or power:weight P_0/W_0 ratio (ie T_0, P_0) based on trend data
    e. Scale 3-view General Arrangement drawing with leading dimensions shown
2. **Initial Baseline Design** (a feasible or workable design) meets all performance requirements
    f. Better estimates of drag polar parameters based on drag buildup, airfoil choices, etc
    g. First-pass modelling of propulsion system performance with speed/altitude/throttle setting
    h. More detailed correlation-based group weight estimation (and mission analysis) for W_0
    i. Constraint analysis to establish W_0/S, T_0/W_0 (or P_0/W_0) that meets all performance requirements
    j. Coupling mission and constraint analysis to fix all of W_0, S and T_0 (or P_0)
3. **Baseline Configuration Development** (optimisation)
    k. Single-parameter optimisation based on dominant performance requirements
    l. Design trades

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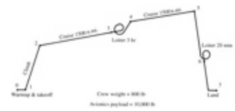
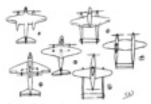
Week 1

Aircraft Design Project Roadmap



RFP: Specified mission, payload and performance requirements.



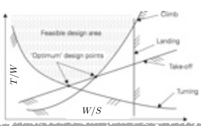
Configuration choice allows first estimates of aero and propulsion properties.

For large passenger transport aircraft this includes passenger cabin layout and considers design family progression.

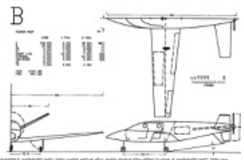
Typically only considers one or two primary mission components (e.g. cruise, landing).

Preliminary sizing based on mission energy (fuel) use, crude empty weight correlations, W/S + T/W historical data.

Typical tools include constraint analysis and carpet plots. Requires moderate computer use to streamline repeated calculations, includes some optimisation.



Week 5



Concept design report with indicative general arrangement drawings

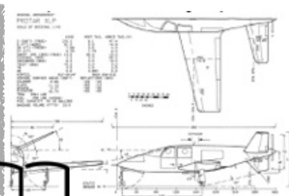
Be prepared to revise earlier decisions and calculations at any stage.

Detailed sizing uses more detailed correlations/ calculations for structural and component weights, aero estimates including drag buildup and polars, sizing of propulsion system to ensure that the design can match all the performance requirements as well as mission requirements.

Sized design with firm choices for all the principal type parameters (W_0, W_e, W_f, T_0, S, A, high-lift system, etc).

Design for operation ties down layout details that ensure the design will work successfully. Includes tail sizing for stability and controllability, balancing the aircraft so the CG is at the required location, final positioning of landing gear, finalising details of wing geometry incl washout distribution and high-lift system. Internal layout details finalised. Indicative structure. Fuel and undercarriage stowage. CAD model. Endpoint: partially optimised workable design.

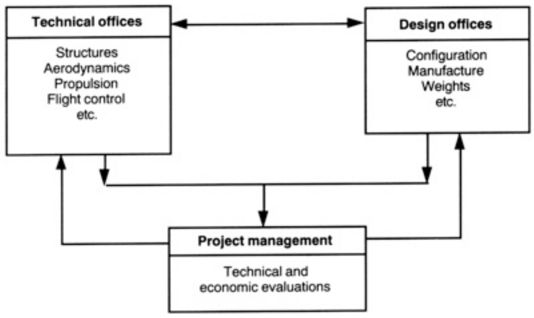
Design report with dimensioned 3-view drawings at a moderate level of detail. All principal design parameters and choices documented and justified. Show performance meets the RFP.



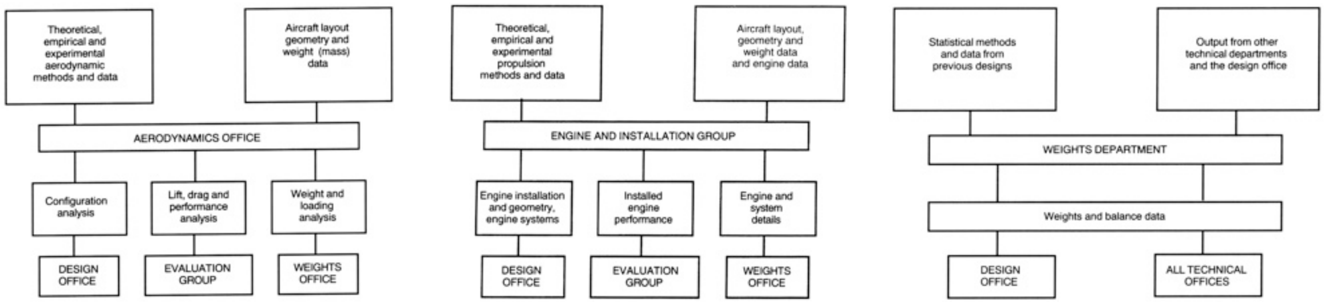
Week 11

Design groups

Aircraft are typically large and complex, and are designed by teams comprised of a number of groups, centrally coordinated.

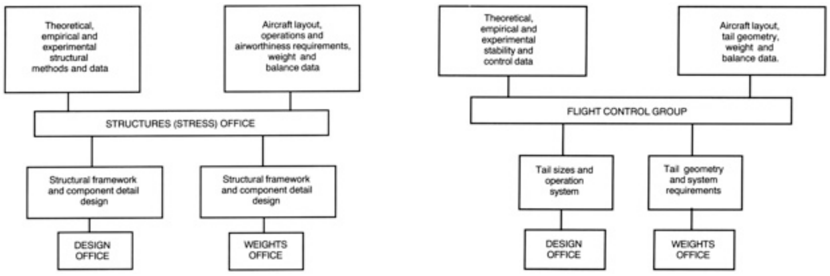


Principal design groups



Selection of other groups

Jenkinson et al.



Texts, reading, reference materials and tools

(Revision):

1. Torenbeek & Wittenberg *Flight Physics*, Springer (2009)

Recommended texts

1. Nicolai & Carichner *Fundamentals of Aircraft and Airship Design,* Vol 1, AIAA (2010)
2. Schaufele *The Elements of Aircraft Preliminary Design*, Aries (2000)
3. Raymer *Aircraft Design, A Conceptual Approach,* 4e, AIAA (2006)
4. Torenbeek *Synthesis of Subsonic Aircraft Design,* DUP/MNP/Springer (1982)

Supplementary texts and reading

6. Shevell, *Fundamentals of Flight*, 2e, Pearson (1989)
7. Brandt, Stiles, Bertin & Whitford *Introduction to Aeronautics: A Design Perspective*, 2e, AIAA (2004)
8. Howe *Aircraft Conceptual Design Synthesis,* PEP/Wiley (2000)
9. Roskam*, Airplane Design Parts I–VIII*, DAR Corp (2005)
10. Hoerner, *Fluid Dynamic Drag,* HFD (1965)
11. Stinton, *The Design of the Aeroplane* and *The Anatomy of the Aeroplane*, Blackwell (1983, 1988)
12. Drela, *Flight Vehicle Aerodynamics*, MIT Press (2014)
13. Brewer, *Hydrogen Aircraft Technology*, CRC Press (1991)

Aircraft layout and analysis software: OpenVSP www.openvsp.org

See also AIAA Design Case Studies: in Books/'Library of Flight' section.

Example design project reports and presentations, Virginia Tech:

http://www.dept.aoe.vt.edu/~mason/Mason_f/SD1SrDesRpts.html

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Aircraft performance assumed background knowledge

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Units and unit conversions

We will typically work in SI units but always in this area one has to convert between British Engineering System and SI units, owing to the fact that international agreements adopted measurement of distance in nautical miles, altitude in feet. Also, many texts use non-SI unit systems.

Quantity	British Engineering System unit	SI unit	Conversion factor (Multiply British Engineering system to get SI value)
Length	ft	m	0.3048
Mile	5280 ft	1.609 km	—
Nautical mile (NM)	6080 ft	1.853 km	—
Area	ft^2	m^2	0.09290
Mass	lbm slug	kg kg	0.4536 14.59
Force	lbf	N	4.448
Pressure and Stress	lbf/ft^2 (psf) $lbf/in.^2$ (psi)	N/m^2 (Pa) kN/m^2 (kPa)	47.88 6.895
Density	lbm/ft^3	kg/m^3	16.02
Temperature difference	°R	K	1/1.8
Specific enthalpy and fuel heating value	Btu/lbm	kJ/kg	2.326
Specific heat (c_p, c_v)	Btu/(lbm-°R)	kJ/(kg-K)	4.187
Gas constant ($g_c R$)	$ft^2/(s^2\text{-}°R)$	$m^2/(s^2\text{-}K)$	0.1672
Rotational speed	rpm	rad/s	$2\pi/60 = 0.1047$
Specific thrust ($F/\dot{m}$)	lbf/(lbm/s)	N-s/kg = m/s	9.807
Thrust specific fuel consumption (S)	$\frac{\text{lbm fuel/h}}{\text{lbf thrust}} = \frac{\text{lbm}}{\text{lbf-h}}$	$\frac{\text{mg fuel/s}}{\text{N thrust}} = \frac{\text{mg}}{\text{N-s}}$	28.33
Power	hp Btu/hr	W W	745.7 0.2931
Power specific fuel consumption (S_P)	$\frac{\text{lbm fuel/h}}{\text{hp}}$	$\frac{\text{mg/s}}{\text{W}} = \frac{\text{mg}}{\text{W-s}}$	0.1690

$1\,\text{kt} = 1\,\text{nm/hr} \approx 100\,\text{ft/min}$
$1\,\text{kt} = 0.5148\,\text{m/s} = 1.853\,\text{km/hr}$

$1\,\text{US gal} = 3.785\,\text{l}$
$1\,\text{UK gal} = 4.546\,\text{l}$

$T(\text{K}) = T(°\text{C}) + 273.16$

Mattingly et al.

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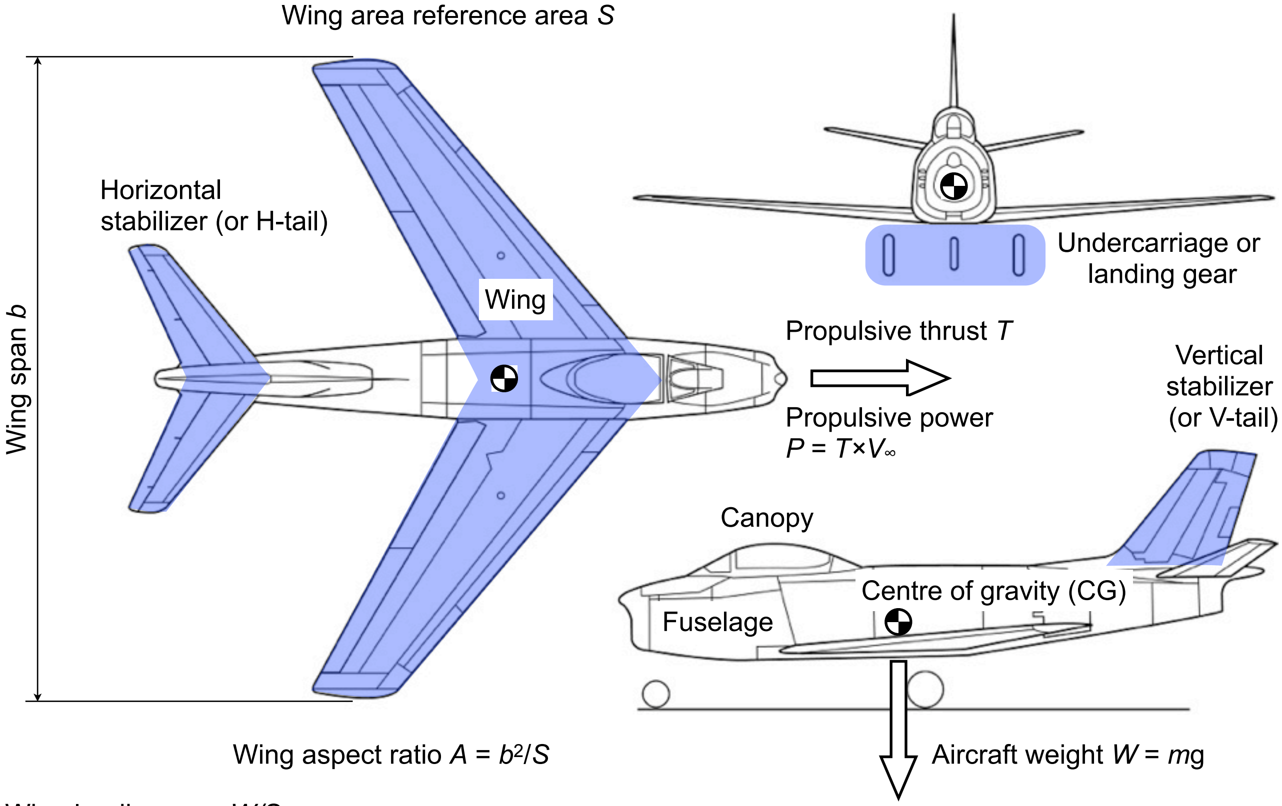
Generally used nomenclature and symbols



Wing loading W/S.

Thrust/weight ratio T/W.

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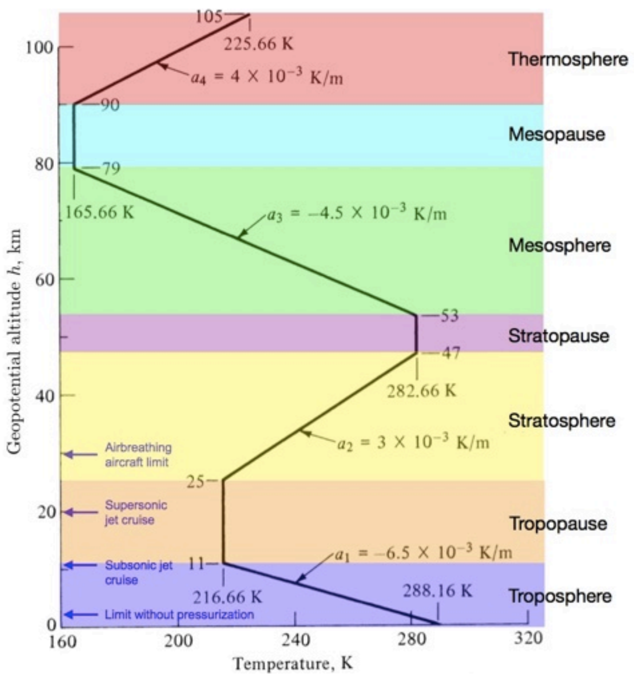
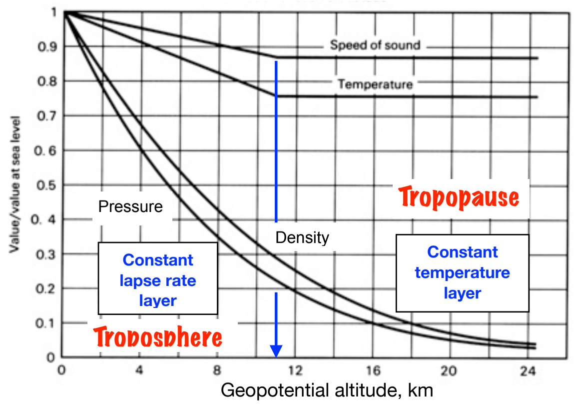
International Standard Atmosphere

Anderson



McCormick



Recall

$\sigma = \rho/\rho_0$

$\delta = p/p_0 \qquad M = V/a$

$\rho = \dfrac{p}{RT}$

a_0	340.3 m/s
p_0	101,325 Pa
ρ_0	1.225 kg/m^3
T_0	288.15 K
R_{air}	287.16 J/kg K
γ_{air}	1.40
g_0	9.8065 m/s^2
r_{Earth}	6.356766×10^6 m

h (km)	T (°C)	a/a_0	$\delta = p/p_0$	$\sigma = \rho/\rho_0$
0	15.0	1.0000	1.0000	1.0000
1	8.5	0.9887	0.8870	0.9075
2	2.0	0.9772	0.7846	0.8217
3	-4.5	0.9656	0.6920	0.7423
4	-11.0	0.9538	0.6085	0.6689
5	-17.5	0.9420	0.5334	0.6012
6	-24.0	0.9299	0.4660	0.5389
7	-30.5	0.9178	0.4057	0.4817
8	-37.0	0.9054	0.3519	0.4292
9	-43.5	0.8929	0.3040	0.3813
10	-50.0	0.8802	0.2615	0.3376
11	-56.5	0.8674	0.2240	0.2978
12	-56.5	0.8671	0.1909	0.2541
13	-56.5	0.8671	0.1632	0.2171
14	-56.5	0.8671	0.1395	0.1856
15	-56.5	0.8671	0.1192	0.1581
16	-56.5	0.8671	0.1019	0.1356
17	-56.5	0.8671	0.0871	0.1159
18	-56.5	0.8671	0.0744	0.0990
19	-56.5	0.8671	0.0636	0.0847
20	-56.5	0.8671	0.0544	0.0724

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Air speeds, Mach & Reynolds numbers

IAS, CAS, EAS, TAS, GS and all that

IAS indicated air speed, read from cockpit instrumentation, includes cockpit instrument error correction;
CAS calibrated air speed, is IAS corrected for airspeed probe position error;
EAS equivalent air speed, is CAS corrected for compressibility effects;
TAS true air speed, is EAS corrected for change in atmospheric density;
GS ground speed, is TAS corrected for (vector) effect of wind speed.

In modern aircraft these conversions all done by onboard computer + GPS.

In design problems we typically use either TAS (symbol V or V_∞) or the equivalent speed at sea level in the standard atmosphere, i.e. EAS (symbol V_e). EAS corrects dynamic pressures for changes in air density to sea level and is convenient in many design calculations (and for pilots).

$\theta = T/T_0$
$\delta = p/p_0$
$\sigma = \rho/\rho_0$
$\sigma = \delta/\theta$

(We generally use the terms IAS and EAS interchangeably.)

$$\tfrac{1}{2}\rho V^2 = \tfrac{1}{2}\rho_0 V_e^2, \quad V_e^2 = \frac{\rho}{\rho_0}V^2 = \sigma V^2, \quad V_e = \sqrt{\sigma}V$$

Mach number

The Mach number M, i.e. TAS divided by speed of sound, a, is a convenient dimensionless speed, especially once compressibility effects become important.

$$a^2 = \frac{\gamma p}{\rho}, \quad \rho = \frac{p}{\mathrm{R}T}, \quad a = \sqrt{\gamma \mathrm{R}T} = a_0\sqrt{\theta}, \quad M = \frac{V}{a} = \frac{V_e}{a_0\sqrt{\delta}}, \quad \tfrac{1}{2}\rho V^2 = \tfrac{1}{2}\gamma p M^2$$

At SL, a_0=340.294m/s, in tropopause (above 11km/36000ft), a=295.06m/s (a and T reduce as h rises).

Reynolds number

The Reynolds number, $Re=\rho Vc/\mu \equiv Vc/\nu$, ($c$ is a reference length, e.g. wing chord) is important for boundary layer effects (e.g. skin friction drag, and $C_{L\max}$).

$\mu = 1.458\times10^{-6}T^{3/2}/(T+110.4)$ Pa.s (where T in K), or approximately $\mu \propto T^{4/5}$, so that $Re \propto \sigma^{1/2}\theta^{-4/5}V_e$.

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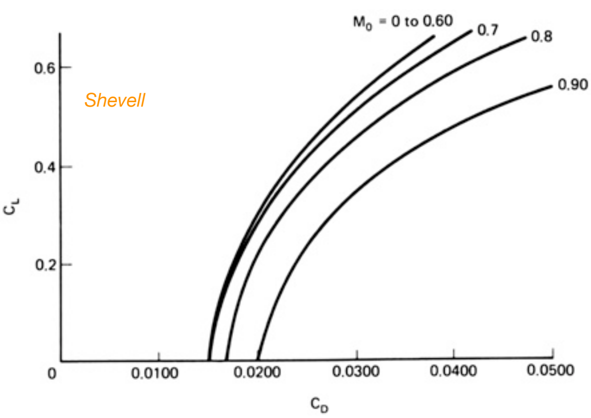
Aircraft drag polar

Aircraft drag comes from a variety of sources, with

- boundary layer skin friction (depends primarily on total 'wetted' surface area S_{wet});
- flow separation (depends on primarily on degree of streamlining, and coefficient of lift, C_L);
- wingtip vortex induced drag (depends on wing aspect ratio A, wing reference area S, and C_L);
- shock wave drag (depends on Mach number, geometry, C_L);

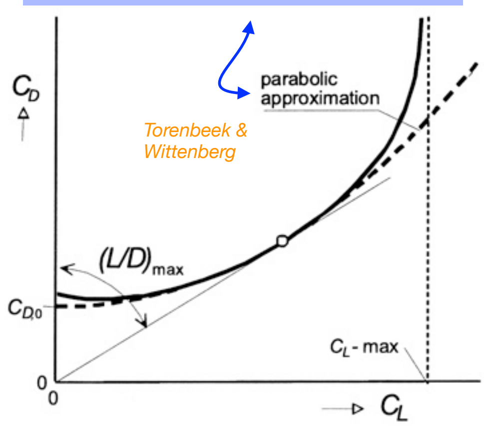
being the dominant contributors to the total sum. In general, all are functions of Reynolds and Mach number and of course aircraft configuration (flaps and undercarriage up/down).



For subsonic/low transonic aircraft, our focus in this course, it is typical to approximate the M = 0 relationship with a simple parabolic function, and make corrections for Mach number effects.

e.g. $$C_D = C_{D,0} + \frac{C_L^2}{\pi A e} \equiv C_{D,0} + KC_L^2$$

For the *speculative design* phase we make estimates of the values C_{D0} and e (or K) in these relationships based on class-typical values but when establishing the *workable design* we compute more precise drag polars.



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Aircraft drag polar

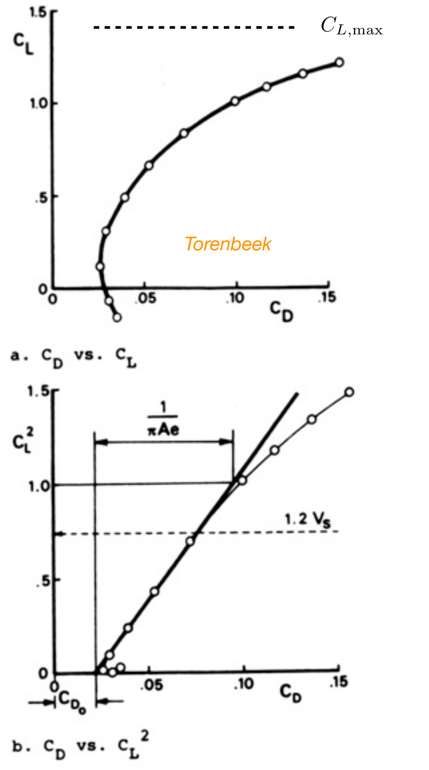

Fig. 5-4. Typical low-speed polar curve

Alternative forms

$$L = \tfrac{1}{2}\rho V^2 S C_L \quad = qSC_L = \tfrac{1}{2}\sigma\rho_0 V^2 S C_L = \tfrac{1}{2}\rho_0 V_e^2 S C_L = \tfrac{1}{2}\gamma p M^2 S C_L$$

$$D = \tfrac{1}{2}\rho V^2 S C_D \approx \tfrac{1}{2}\rho V^2 S\left(C_{D,0} + \frac{C_L^2}{\pi A e}\right) = qS(C_{D,0} + C_{D,i})$$

$C_{D,0}$ is often called the 'zero-lift drag coefficient' (but strictly, isn't) and *e* is called the 'aircraft (or Oswald), efficiency factor'.

More correctly, both $C_{D,0}$ and *e* are parameters in a simple parabolic approximation (curve fit) to the true drag polar.

This parabolic approximation is usually adequate at the conceptual design phase, especially if the aircraft's design C_L value is not large.

Representative values:

	C_{D_o}	e
high-subsonic jet aircraft	.014 - .020	.75 - .85*
large turbopropeller aircraft	.018 - .024	.80 - .85
twin-engine piston aircraft	.022 - .028	.75 - .80
small single engine aircraft		
retractable gear	.020 - .030	.75 - .80
fixed gear	.025 - .040	.65 - .75
agricultural aircraft:		
- spray system removed	.060	.65 - .75
- spray system installed	.070 - .080	.65 - .75

Torenbeek

$C_{D,0}$ may be rationally estimated in a 'drag breakdown' of individual component drag coefficient and reference area values:

$$C_{D,0} = \frac{\sum_j C_{D,j} S_j}{S}$$

Essentially this amounts to summing all the individual drag forces and normalizing the total based on the wing reference area *S*.

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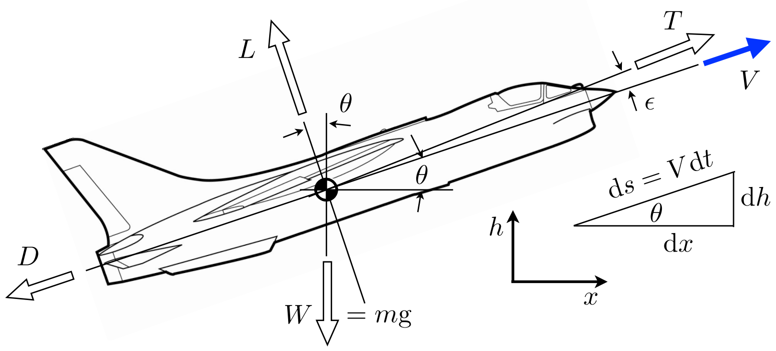
Aircraft equations of motion



First consider flight where there is no bank angle, and all forces and motion occur in the aircraft plane of symmetry.

The aircraft is assumed to be trimmed so that moments sum to zero.

From flight path geometry we have

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \dot{x} = V\cos\theta$$

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \dot{h} = V\sin\theta$$

Newton's second law is $\sum \boldsymbol{F} = \dfrac{\mathrm{d}(m\boldsymbol{V})}{\mathrm{d}t} = m\dfrac{\mathrm{d}\boldsymbol{V}}{\mathrm{d}t} + \boldsymbol{V}\dfrac{\mathrm{d}m}{\mathrm{d}t} = m\dot{\boldsymbol{V}} + \boldsymbol{V}\dot{m} \approx m\dot{\boldsymbol{V}}$

Note that it is usual in aircraft performance dynamics to ignore the time rate of change of mass, except when computing fuel consumption. This assumption may be inadequate when fuel burn rates are high, e.g. for performance analysis of missiles.

Now we consider components tangential and normal to the flight path, leading to

Tangential $\quad m\dfrac{\mathrm{d}V}{\mathrm{d}t} = T\cos\epsilon - D - mg\sin\theta = m\dot{V}$

Normal $\quad mV\dfrac{\mathrm{d}\theta}{\mathrm{d}t} = T\sin\epsilon + L - mg\cos\theta = mV\dot{\theta} \equiv m\dfrac{V^2}{R_V}$ $\quad$ where R_V is flight path radius of curvature.

(Divide tangential equation through by *mg* = *W*.)

Rearrange:

(Divide normal equation through by *mgV*.)

$$\frac{\dot{V}}{g} = \frac{T\cos\epsilon - D}{W} - \sin\theta$$

$$\frac{\dot{\theta}}{g} = \frac{T\sin\epsilon + L}{VW} - \frac{\cos\theta}{V}$$

Aircraft equations of motion

Since the thrust and lift are functions of altitude and speed, the drag is additionally a function of lift, and the fuel burn rate depends on thrust, we have a set of five ODEs to consider, four of which are coupled:

Integrate to get range $\dot{x} = V\cos\theta$ (not directly coupled to the remaining four)

Note coupling of terms between equations: cannot solve them independently

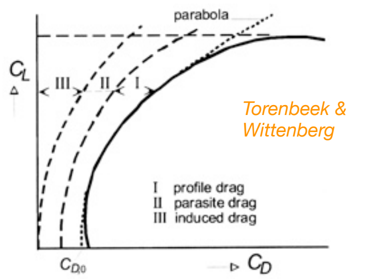
Integrate to get *h* $\dot{h} = V\sin\theta$

Integrate to get *W* $\dot{W} = -\mathrm{g}c_t(h,V)T(h,V)$

Integrate to get *V* $\dfrac{\dot{V}}{\mathrm{g}} = \dfrac{T(h,V)\cos\epsilon - D(h,V,L)}{W} - \sin\theta$

Integrate to get *θ* $\dfrac{\dot{\theta}}{\mathrm{g}} = \dfrac{T(h,V)\sin\epsilon + L(h,V)}{VW} - \dfrac{\cos\theta}{V}$

Drag polar $C_D = C_{D,0} + KC_L^2$



The standard approach for most performance analysis is to obtain decoupling by assuming that θ=const., and base everything around the 4th equation in the set. i.e. the one containing the drag polar. (Also, since ϵ is typically small or zero, cosϵ→1 and sinϵ→ϵ.) Thus, starting with the 4th equation,

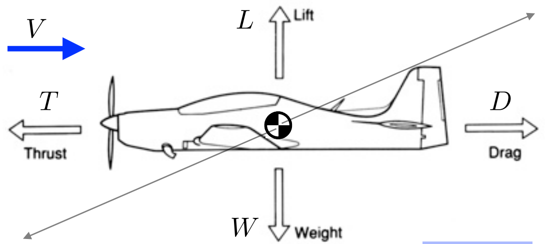
$$\frac{(T-D)V}{W} = V\sin\theta + V\frac{\dot{V}}{\mathrm{g}} = \frac{\mathrm{d}}{\mathrm{d}t}\left(h + \frac{V^2}{2\mathrm{g}}\right) = \frac{\mathrm{d}e}{\mathrm{d}t}$$

Fundamental Performance Equation

where *e*, the sum of potential and kinetic energies per unit weight, is called the aircraft's specific energy or energy height. The equation is often called the Fundamental Performance Equation and is the basis for most performance analysis (and design for performance). The rate of change of KE is often ignored.

The term (*T*–*D*)/*W* is called the specific excess thrust and (*T*–*D*)*V*/*W* is called the specific excess power, i.e. the amount of thrust/power per unit weight available to increase the aircraft's altitude or speed, or both.

Aircraft performance in steady, level flight



Returning to

$$\frac{\dot{V}}{\mathrm{g}} = \frac{T\cos\epsilon - D}{W} - \sin\theta$$

$$\frac{\dot{\theta}}{\mathrm{g}} = \frac{T\sin\epsilon + L}{VW} - \frac{\cos\theta}{V}$$

with $\dot{V} = 0,\quad \theta = 0,\quad \dot{\theta} = 0$ and $\epsilon = 0$

We have the simple relationships

$$T = D$$
$$W = L$$

From similar triangles

$$\frac{T}{W} = \frac{D}{L} = \frac{C_D}{C_L} = \frac{1}{C_L/C_D}$$

This is the fundamental performance equation in its most simplified form: $\dfrac{T-D}{W} = 0$

Thrust required to fly: $T_R = \dfrac{W}{C_L/C_D}$ Minimum thrust required to fly: $T_{R,\min} = \dfrac{W}{(C_L/C_D)_{\max}}$

We now look at the relationship between thrust required (drag) and speed, given *W*, *S* and altitude (*ρ*).

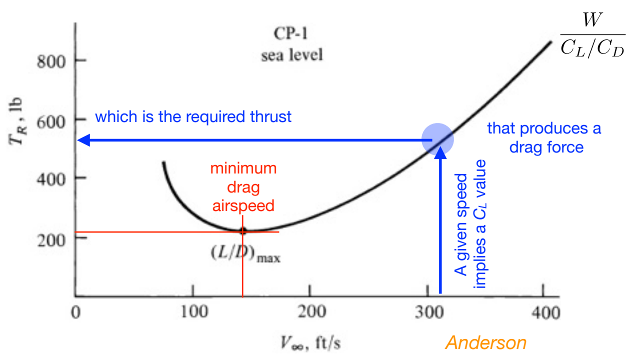
❶ $L = \frac{1}{2}\rho V^2 S C_L = W$

❷ $C_L = \dfrac{1}{\frac{1}{2}\rho V^2}\dfrac{W}{S} = \dfrac{2}{\rho}\dfrac{W}{S}\dfrac{1}{V^2}$

❸ $C_D = C_{D,0} + KC_L^2$

❹ $\dfrac{C_L}{C_D} = \dfrac{C_L}{C_{D,0} + KC_L^2}$

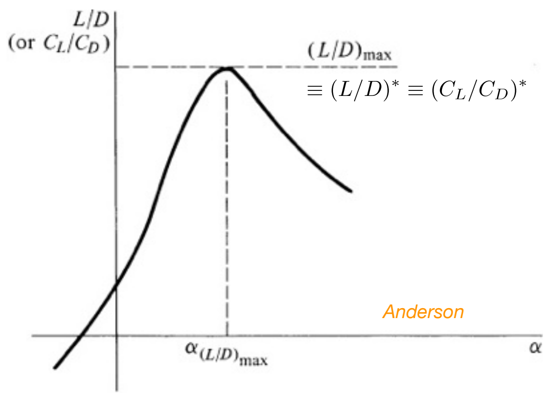
❺ $T = D = \dfrac{W}{C_L/C_D}$

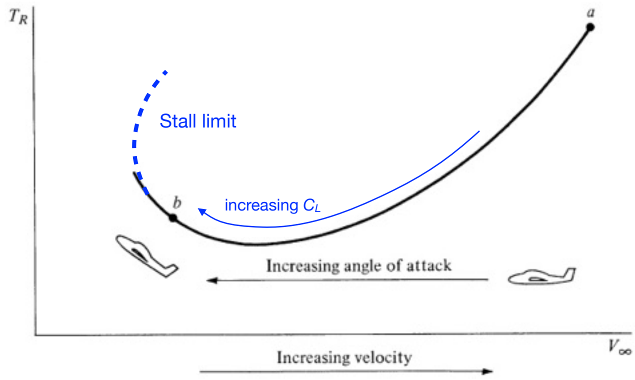


We find there is a definite speed corresponding to minimum drag and for larger thrusts there could be two possible speeds.

Lift, drag & thrust in level flight

The underlying reason is that the lift/drag ratio depends on angle of attack and C_L through the speed of flight.





Through some analysis

$$D = q_\infty S C_D = q_\infty S\left(C_{D,0} + K C_L^2\right)$$

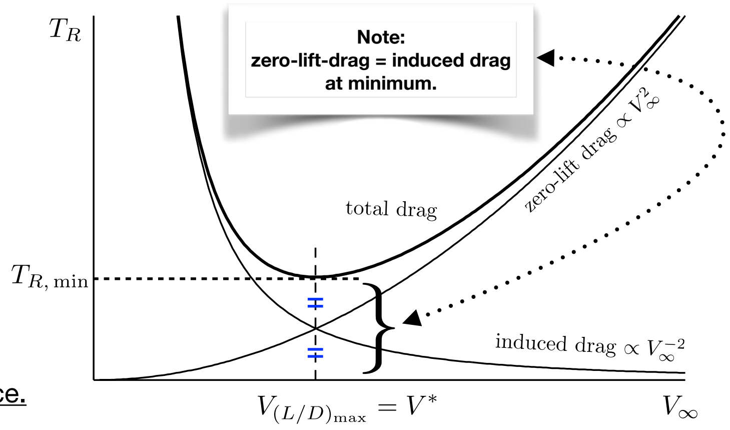
$$C_L = \frac{W}{q_\infty S}$$

$$T_R = D = S\left[\tfrac{1}{2}\rho V_\infty^2 C_{D,0} + \frac{K}{\tfrac{1}{2}\rho V_\infty^2}\left(\frac{W}{S}\right)^2\right]$$

zero-lift drag   lift-induced drag

we see that one contribution to drag varies with V^2, the other with V^{-2}.

Understanding and manipulating these relationships is the key to aircraft performance.



Lift, drag & thrust in level flight

Coefficient of lift at L/D_{max}, $C_L{}^*$

$$\frac{C_D}{C_L} = \frac{C_{D,0} + K C_L^2}{C_L} = \frac{C_{D,0}}{C_L} + K C_L$$

Find TP: $\dfrac{\mathrm{d}(C_D/C_L)}{\mathrm{d}C_L} = -\dfrac{C_{D,0}}{C_L^2} + K = 0$

i.e. $K = \dfrac{C_{D,0}}{C_L^{*2}}$

$$C_L^* = \sqrt{\frac{C_{D,0}}{K}}$$

$$C_D^* = 2C_{D,0}$$

Corresponding L/D_{max}, $(C_L/C_D)^*$

$$\left(\frac{C_L}{C_D}\right)^* = \frac{C_L^*}{C_{D,0} + K C_L^{*2}} = \frac{C_L^*}{C_{D,0} + K(C_{D,0}/K)} = \frac{C_L^*}{2C_{D,0}} = \frac{\sqrt{C_{D,0}}}{\sqrt{K}2C_{D,0}} = \frac{1}{\sqrt{4C_{D,0}K}}$$

Minimum-drag airspeed in level flight, V^*

$$V^* = \sqrt{\frac{2}{\rho}\frac{W}{S}\frac{1}{C_L^*}} = \left(\frac{2}{\rho}\frac{W}{S}\right)^{1/2}\left(\frac{K}{C_{D,0}}\right)^{1/4}$$

Dimensionless airspeed, u

$$V_\infty \equiv V = uV^*$$

$$\tfrac{1}{2}\rho V_\infty = \tfrac{1}{2}\rho V_\infty^{*2} u^2$$

so

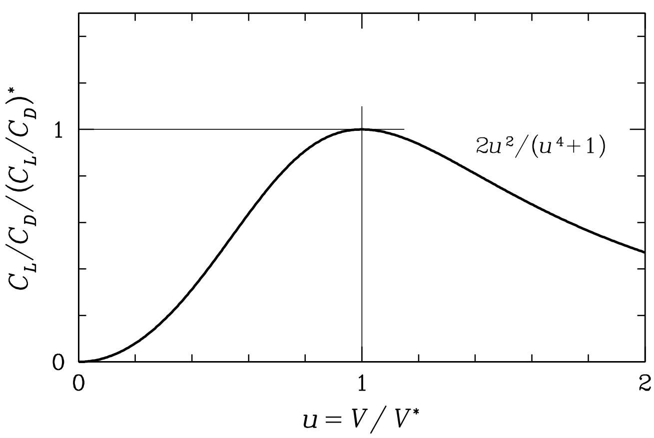
$$C_L = \frac{2}{\rho}\frac{W}{S}\frac{1}{V^{*2}}\frac{1}{u^2} = C_L^*\frac{1}{u^2}$$

Dependence of L/D on airspeed

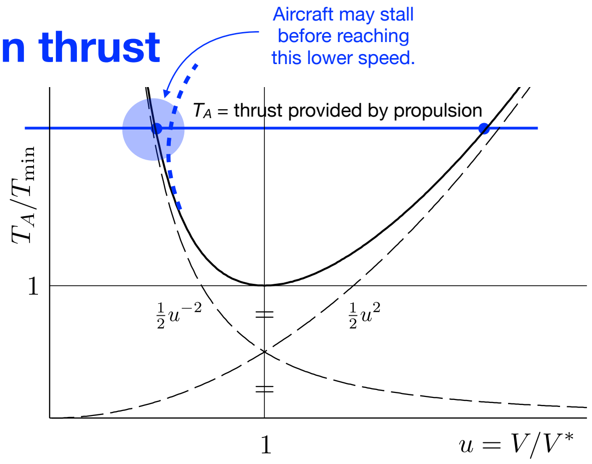
$$\frac{C_L}{C_D} = \frac{C_L^*/u^2}{C_{D,0} + K C_L^{*2}/u^4} = \frac{C_L^*(1/u^2)}{C_{D,0}\left(1 + 1/u^4\right)}$$

$$\frac{C_L}{C_D} = \frac{2}{2\sqrt{C_{D,0}K}}\frac{u^2}{u^4+1} = \left(\frac{C_L}{C_D}\right)^*\frac{2u^2}{u^4+1}$$

Recall $T_R = \dfrac{W}{C_L/C_D}$ now we have a clearer understanding of how T_R depends on speed.



Speed for a given thrust



What is the speed for a propulsive thrust $T_A > T_{\min}$?

We see that there are two possible solutions.

$$\frac{T_A}{W} = \frac{D}{W} = \frac{D}{L} = \left(\frac{C_D}{C_L}\right)^* \frac{u^2 + u^{-2}}{2} = \left(\frac{C_D}{C_L}\right)^* \frac{u^4 + 1}{2u^2}$$

Rearrange:

$$u^4 - 2\frac{T_A}{W}\left(\frac{C_L}{C_D}\right)^* u^2 + 1 = 0$$

$$x^2 + px + q = 0$$

$$x = -\frac{p}{2} \pm \sqrt{\left(\frac{p}{2}\right)^2 - q}$$

Solve quadratic in u^2:

$$u^2 = \frac{T_A}{W}\left(\frac{C_L}{C_D}\right)^* \pm \left(\left[\frac{T_A}{W}\left(\frac{C_L}{C_D}\right)^*\right]^2 - 1\right)^{1/2}$$

$$u_{1,2} = \left\{\frac{T_A}{W}\left(\frac{C_L}{C_D}\right)^* \pm \left(\left[\frac{T_A}{W}\left(\frac{C_L}{C_D}\right)^*\right]^2 - 1\right)^{1/2}\right\}^{1/2}$$

One can show that $u_1 u_2 = 1$

TAS $$V = V^* u = \left(\frac{2}{\rho}\frac{W}{S}\frac{1}{C_L^*}\right)^{1/2} u$$

EAS $$V_e = V_e^* u = \sqrt{\sigma} V^* u$$ where $$\sigma = \frac{\rho}{\rho_{SL}}$$

Alternatively, the equivalent without exploiting non-dimensionalization w.r.t. the minimum-drag values is:

$$V_{1,2} = \left\{\frac{(T_A/W)(W/S) \pm (W/S)\left([T_A/W]^2 - 4C_{D,0}K\right)^{1/2}}{\rho\, C_{D,0}}\right\}^{1/2}$$

Altitude effect on thrust required

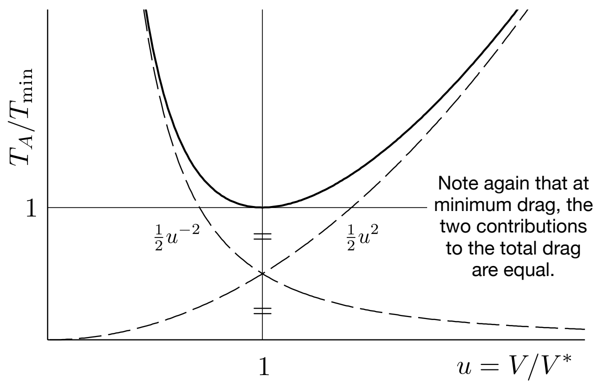
How does a change in altitude affect the thrust required?

$$\left(\frac{T}{W}\right)_{\min} = \left(\frac{D}{L}\right)^* = 2\sqrt{C_{D,0}K}, \quad T_{\min} = 2W\sqrt{C_{D,0}K}$$

Note that this does not depend on altitude.
Reducing W, $C_{D,0}$ or K reduces $T_{\min}$.

We see that for this condition, the zero-lift drag equals the induced drag:

$$C_D^* = C_{D,0} + KC_L^{*2} = C_{D,0} + KC_{D,0}/K = 2C_{D,0}$$



How does a change in altitude affect the speed?

EAS

$$V_e^* = \left(\frac{2}{\rho_0}\frac{W}{S}\right)^{1/2}\left(\frac{1}{C_L^*}\right)^{1/2} = \left(\frac{2}{\rho_0}\frac{W}{S}\right)^{1/2}\left(\frac{K}{C_{D,0}}\right)^{1/4}$$

(a constant, SL-equivalent value).

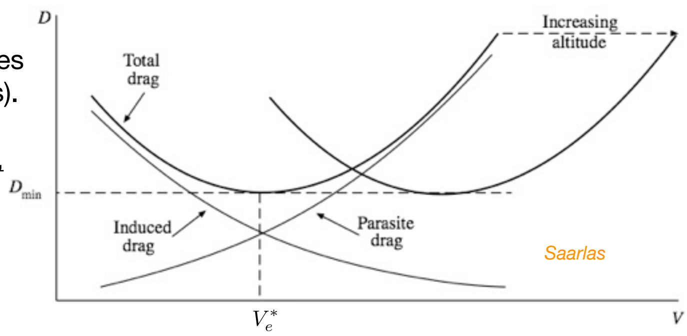
TAS

$$V^* = \left(\frac{2}{\rho}\frac{W}{S}\right)^{1/2}\left(\frac{1}{C_L^*}\right)^{1/2} = \frac{V_e^*}{\sqrt{\sigma}}$$

(recall σ reduces as h increases).

★ The minimum drag force is independent of altitude, but the corresponding TAS increases with altitude.

Provided there is enough thrust, we can fly faster by increasing altitude (or increasing wing loading).



Range (distance flown) and endurance (time flown)

First, the simple principles:

Two idealized engine classes:

1. fuel mass flow rate $\dot{m}_f$ is proportional to power (typical of **prop**eller engine incl. turboprop);
2. fuel mass flow rate $\dot{m}_f$ is proportional to thrust (typical of **jet** engine).

To maximize endurance, minimize the weight of fuel consumed per unit time, i.e. $\dot{m}_f$

To maximize range, minimize the weight of fuel consumed per unit distance, i.e. $\dot{m}_f \Delta t / \Delta s = \dot{m}_f / V_\infty$

Recall:

For power-type propulsion we use power-specific fuel consumption, PSFC, given the symbol c_p.

SI units are kg/W.s. Note the values below are in mg/W.s

Typical PSFCs, c_p: lbm/bhp/hr {mg/W.s}	(Range) Cruise	(Endurance) Loiter
Piston-prop (fixed pitch)	0.4 {.068}	0.5 {.085}
Piston-prop (variable pitch)	0.4 {.068}	0.5 {.085}
Turboprop	0.5 {.085}	0.6 {.101}

For thrust-type propulsion we use thrust-specific fuel consumption, TSFC, given the symbol c_t.

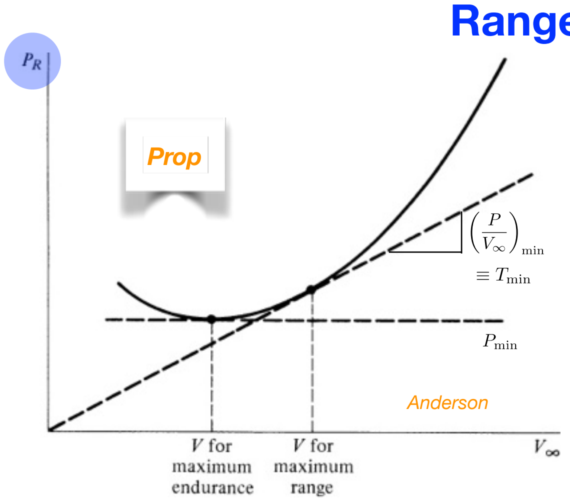
SI units are kg/N.s. Note the typical values below are in mg/N.s

Typical TSFCs, c_t: lbm/lbf/hr {mg/N.s}	Cruise	Loiter
Pure turbojet	0.9 {25.5}	0.8 {22.7}
Low-bypass turbofan	0.8 {22.7}	0.7 {19.8}
High-bypass turbofan	0.5 {14.1}	0.4 {11.3}

Raymer

NB: all the above values are for hydrocarbon fuels - convert to other types based on implied power consumptions?

Range and endurance



Propeller+piston, best characterized by power output

PSFC c_p = mass of fuel consumed per unit power per unit time.

Maximum endurance: minimise kg of fuel per second

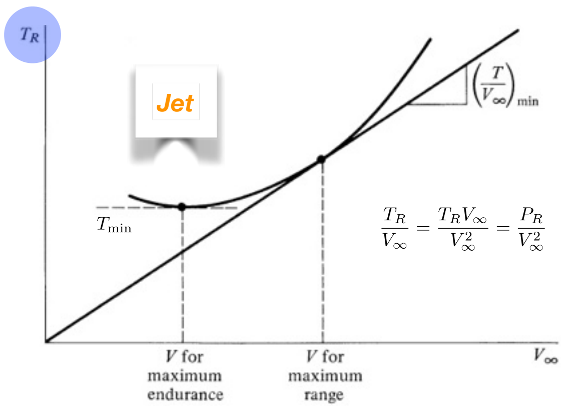
kg fuel/second ∝ $c_p \times P_R$.

Fly at minimum-power speed, i.e. $(C_L^{3/2}/C_D)_{max}$.

Maximum range: minimize kg of fuel per m travelled

kg fuel/m ∝ (kg fuel/sec)/(m/sec) = $c_p \times P_R / V_\infty = c_p \times T_R$.

Fly at minimum-drag (and thrust) speed, V^*, $(C_L/C_D)_{max}$.



Jet, best characterized by thrust output

TSFC c_t = mass of fuel consumed per unit thrust per unit time.

Maximum endurance: minimise kg of fuel per second

kg fuel/second ∝ $c_t \times T_R$.

Fly at minimum-drag (and thrust) speed, V^*, $(C_L/C_D)_{max}$.

Maximum range: minimize kg of fuel per m travelled

kg fuel/m ∝ (kg fuel/sec)/(m/sec) = $c_t \times T_R / V_\infty$.

Now $V_\infty = \sqrt{\frac{2}{\rho}\frac{W}{S}\frac{1}{C_L}}$ so $\frac{T_R}{V_\infty} = \frac{W}{V_\infty (C_L/C_D)} \propto \frac{1}{C_L^{1/2}/C_D}$

Fly at minimum power/kinetic energy speed, i.e. $(C_L^{1/2}/C_D)_{max}$.

Range and endurance

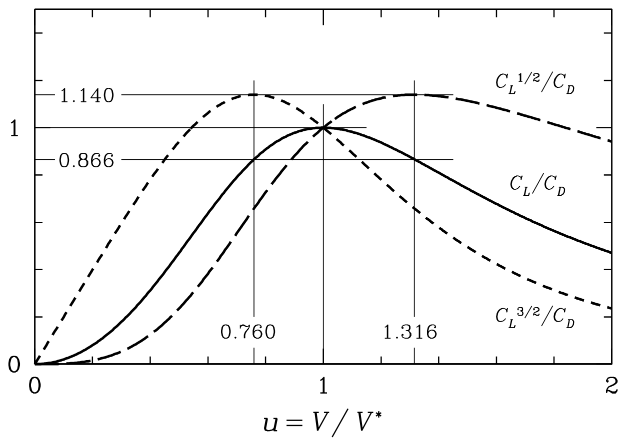
The ratios $C_L^{3/2}/C_D \propto L/(DV)$, $C_L/C_D \propto L/D$ and $C_L^{1/2}/C_D \propto VL/D$ can all be considered as functions of dimensionless speed $u=V/V^*$ where V^* is the minimum-drag speed.

We already know

$$\frac{L}{D} = \left(\frac{C_L}{C_D}\right)^* \frac{2}{u^2+u^{-2}} = \left(\frac{C_L}{C_D}\right)^* \frac{2u^2}{u^4+1}$$

and $$\frac{L}{DV} \propto \frac{1}{u}\frac{2u^2}{u^4+1} = \frac{2u}{u^4+1}$$ Speed function associated with best prop endurance

so $$\frac{VL}{D} \propto u\frac{2u^2}{u^4+1} = \frac{2u^3}{u^4+1}$$ Speed function associated with best jet range



Plotting and then analysing these functions we can find various useful ratios associated with the maxima which are tabulated below.

Function	***Dimensionless***	***V/V*, at max***	**(L/D)/(L/D)*, at max**	**C_L/C_L^*, at max**
$L/(DV)$	$C_L^{3/2}/C_D$	$(1/3)^{1/4} = 0.760$	$(3/4)^{1/2} = 0.866$	$3^{1/2} = 1.732$
L/D	C_L/C_D	1	1	1
$(VL)/D$	$C_L^{1/2}/C_D$	$(3)^{1/4} = 1.316$	$(3/4)^{1/2} = 0.866$	$(1/3)^{1/2} = 0.577$

Recall: $C_L^* = (C_{D,0}/K)^{1/2}$; $(C_L/C_D)^* = 1/(4C_{D,0}K)^{1/2}$; $V^* = [(2/\rho)(W/S)(1/C_L^*)]^{1/2}$.

Range and endurance

<u>**Range**; based on weight of fuel consumed per unit distance.</u> Minimize $\dot{m}_{\text{fuel}}\mathrm{d}t/\mathrm{d}x = \dot{m}_{\text{fuel}}/V_\infty$

$$\dot{m}g\frac{\mathrm{d}t}{\mathrm{d}x} = -\frac{\dot{W}}{V} = -\frac{\mathrm{d}W}{\mathrm{d}t}\frac{\mathrm{d}t}{\mathrm{d}x} = -\frac{\mathrm{d}W}{\mathrm{d}x} \equiv -\frac{\mathrm{d}W}{\mathrm{d}R}$$ where R is range. Hence $$\frac{\mathrm{d}R}{\mathrm{d}W} = -\frac{V}{\dot{m}g}$$

Jet $$\dot{m}g = \mathrm{g}\,c_t\,T \quad \text{and} \quad T = \frac{W}{L/D}$$

$$\frac{\mathrm{d}R}{\mathrm{d}W} = -\frac{V}{\dot{m}g} = -\frac{V}{\mathrm{g}c_t T} = -\frac{1}{\mathrm{g}c_t}V\frac{L}{D}\frac{1}{W}$$ i.e. $$R = -\int_{W_i}^{W_f}\frac{1}{\mathrm{g}c_t}V\frac{L}{D}\frac{\mathrm{d}W}{W} = \int_{W_f}^{W_i}\frac{1}{\mathrm{g}c_t}V\frac{L}{D}\frac{\mathrm{d}W}{W}$$

where W_i is the initial, W_f is the final aircraft weight for a flight segment.

Now <u>assuming</u> c_t, V, L/D are all constants: $$R = \frac{1}{\mathrm{g}c_t}V\frac{L}{D}\ln\frac{W_i}{W_f}$$

We already know that VL/D is largest when we fly at $(C_L^{1/2}/C_D)_{\max}$., i.e. $L/D=(3/4)^{1/2}(L/D)^*$ and $V=3^{1/4}V^*$.

$$R_{\max} = \left(\frac{27}{16}\right)^{1/4}\frac{1}{\mathrm{g}c_t}V^*\left(\frac{L}{D}\right)^*\ln\frac{W_i}{W_f} = \frac{1.140}{\mathrm{g}c_t}V^*\left(\frac{L}{D}\right)^*\ln\frac{W_i}{W_f}$$

Prop $$\dot{m}g = \mathrm{g}c_p P = \frac{\mathrm{g}c_p DV}{\eta_{\text{pr}}} = \frac{\mathrm{g}c_p TV}{\eta_{\text{pr}}} = \frac{\mathrm{g}c_p WV}{\eta_{\text{pr}}(L/D)} \qquad \frac{\mathrm{d}R}{\mathrm{d}W} = -\frac{V}{\dot{m}g} = -\frac{\eta_{\text{pr}}(L/D)}{\mathrm{g}c_p W}$$

η_{pr} is propeller efficiency, $\eta_{pr} \approx 0.8$.

$$\mathrm{d}R = -\frac{\eta_{\text{pr}}}{\mathrm{g}c_p}\frac{L}{D}\frac{\mathrm{d}W}{W}$$ Hence, to maximise, assuming L/D and c_p const: $$R_{\max} = \frac{\eta_{\text{pr}}}{\mathrm{g}c_p}\left(\frac{L}{D}\right)^*\ln\frac{W_i}{W_f}$$

(As originally derived by French engineer Breguet – the generic label for all these related equations.)

Range and endurance

Endurance; based on weight of fuel consumed per unit time. Minimize $\dot{m}_{\text{fuel}}$

$$\dot{m}g = -\dot{W} = -\frac{\mathrm{d}W}{\mathrm{d}t} \equiv -\frac{\mathrm{d}W}{\mathrm{d}E}$$ where E is endurance. Hence $$\frac{\mathrm{d}E}{\mathrm{d}W} = -\frac{1}{\dot{m}g}$$

Jet $\dot{m}g = g\,c_t\,T$ Then, working similarly to previously for jet range, we have

$$\mathrm{d}E = -\frac{1}{gc_t}\frac{L}{D}\frac{\mathrm{d}W}{W}$$ Assuming c_t and L/D const: $$E = \frac{1}{gc_t}\frac{L}{D}\ln\frac{W_i}{W_f}$$ $$E_{\max} = \frac{1}{gc_t}\left(\frac{L}{D}\right)^{*}\ln\frac{W_i}{W_f}$$

Prop As above: $$\dot{m}g = gc_pP = \frac{gc_pDV}{\eta_{\text{pr}}} = \frac{gc_pTV}{\eta_{\text{pr}}} = \frac{gc_pWV}{\eta_{\text{pr}}(L/D)}$$

$$\mathrm{d}E = -\frac{\eta_{\text{pr}}}{gc_p}\frac{1}{V}\frac{L}{D}\frac{\mathrm{d}W}{W} \qquad E = \frac{\eta_{\text{pr}}}{gc_p}\frac{1}{V}\frac{L}{D}\ln\frac{W_i}{W_f}$$ which we know is maximized when flying at $(C_L^{3/2}/C_D)_{\max}$,

where $L/D = 0.866(L/D)^*$, $V = 0.760V^*$: $$E_{\max} = \left(\frac{27}{16}\right)^{1/4}\frac{\eta_{\text{pr}}}{gc_p}\frac{1}{V^*}\left(\frac{L}{D}\right)^{*}\ln\frac{W_i}{W_f}$$

NB: In a design problem, we will often have the range or endurance specified and need to find the weight fraction W_i/W_f and from this $W_{\text{fuel}} = W_i(1-W_f/W_i)$.

To maximize endurance for propeller aircraft, typically want to have small induced drag (hence high aspect ratio), and fly at low altitude. This **Breguet** Atlantique ASW aircraft is one example.

Range and endurance

Summary

Optimal conditions for range and endurance all demand flying at fixed points on the drag polar.

Range, *R*

Type	Equation	Optimal flight strategy
Jet	$R = \frac{1}{gc_t}V\frac{L}{D}\ln\frac{W_i}{W_f}$	Maximized at high altitude (high V ⇒ low ρ). Fly at $1.316V^*$, $0.577C_L^*$, $0.866(C_L/C_D)^*$.
Prop	$R = \frac{\eta_{\text{pr}}}{gc_p}\frac{L}{D}\ln\frac{W_i}{W_f}$	Independent of altitude. Fly at V^*, C_L^*, $(C_L/C_D)^*$.

A conceptual difficulty is that for jet aircraft, the above suggests that range increases indefinitely as altitude is increased (reducing ρ, increasing V^*). Eventually, Mach number and/or propulsion system limits start to take effect and invalidate this simple model. But for now, it's good enough.

Endurance, *E*

Type	Equation	Optimal flight strategy
Jet	$E = \frac{1}{gc_t}\frac{L}{D}\ln\frac{W_i}{W_f}$	Independent of altitude. Fly at V^*, C_L^*, $(C_L/C_D)^*$.
Prop	$E = \frac{\eta_{\text{pr}}}{gc_p}\frac{1}{V}\frac{L}{D}\ln\frac{W_i}{W_f}$	Maximized at low altitude (low V ⇒ high ρ). Fly at $0.760V^*$, $1.732C_L^*$, $0.866(C_L/C_D)^*$.

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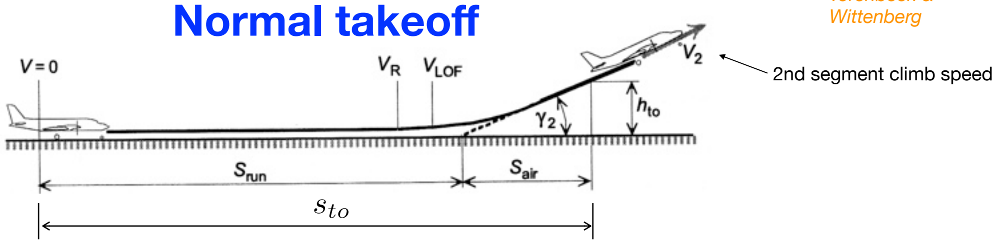
Normal takeoff

$$s_{to} \approx \frac{V_2^2}{2\bar{a}} + \frac{h_{to}}{\tan\gamma_2}$$

s_{run} s_{air}

$$\bar{a} = r_T \frac{T_{to}}{W} g$$

$$r_T \approx 0.8 - 0.9 \quad \text{(jet)}$$



Recall for steady climb $\sin\gamma_2 = \frac{T-D}{W} = \frac{T}{W} - \frac{D}{W}$ where $W \to L = \frac{1}{2}\rho V_2^2 S C_{L_2}$ or $V_2^2 = \frac{2}{\rho}\frac{W}{S}\frac{1}{C_{L_2}}$

$$\approx \left(\frac{T}{W} - \frac{C_D}{C_L}\right)_2$$

$C_{L_2} \approx C_{L,\max}/k_{to}^2$ and $C_D = C_{D,0} + K C_{L_2}^2$

$k_{to} \approx 1.2$

and $\sin\gamma_2 \longrightarrow \tan\gamma_2$

finally

$$s_{to} \approx \frac{1}{\rho g r_T}\frac{1}{C_{L_2}}\frac{W}{S}\frac{W}{T_{to}} + \frac{h_{to}}{T_2/W - (C_D/C_L)_2}$$

s_{run} s_{air}

Typically an additional 15% safety margin is added to this value (or any better estimate).

Note that T_2 may be significantly smaller than T_{to}, especially so for a propeller aircraft.

1. Ground run s_{run} increases quadratically with weight W and is reduced by either decreasing the wing loading W/S or increasing the thrust/weight ratio T_{to}/W, or both. Increasing weight also increases the air distance s_{air}.
2. Air density+temperature may have a significant effect on both ρ and T. High, hot take-offs are worst.
3. Increasing flap deflections will increase $C_{L\max}$ and hence C_{L2}, which reduces the first term but increases C_D/C_L and hence the second term. There is an optimum flap deflection which is typically less than the value used at landing, and hence $C_{L,\max,to} < C_{L,\max,land}$.

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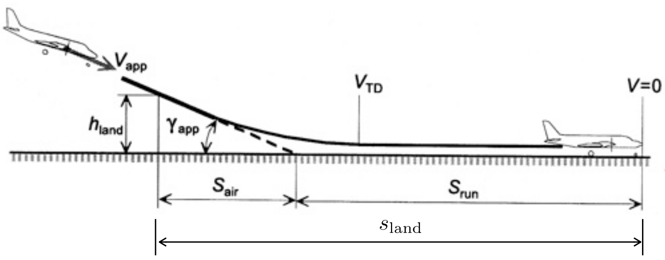
Landing



Torenbeek & Wittenberg

1. During *landing approach* the aircraft flies at a steady speed $V_{app} > k_{app} V_{stall} = 1.3\, V_{stall}$. The gradient γ_{app} is typically around 2.5° to 3° for commercial aircraft.
2. Once the *runway threshold height* h_{land} (typically 50 ft or 15 m) is reached the engines are throttled back and the pilot executes a *landing flare* or *round-out* to touch-down at V_{td}, typically 1.15 V_{stall}.
3. After touch-down of all the undercarriage elements the aircraft is slowed by wheel brakes (and perhaps airbrakes) until it comes to rest. While engine thrust reversal may be applied this is typically not included in an analysis designed to compute the minimum runway length required.

For a simplified analysis, $s_{land} = s_{air} + s_{run} \approx \frac{h_{land}}{\tan\gamma_{app}} + \frac{V_{app}^2}{2|\bar{a}|}$ where $V_{app} = 1.3\sqrt{\frac{2}{\rho}\frac{W}{S}\frac{1}{C_{L,\max}}}$

leading to

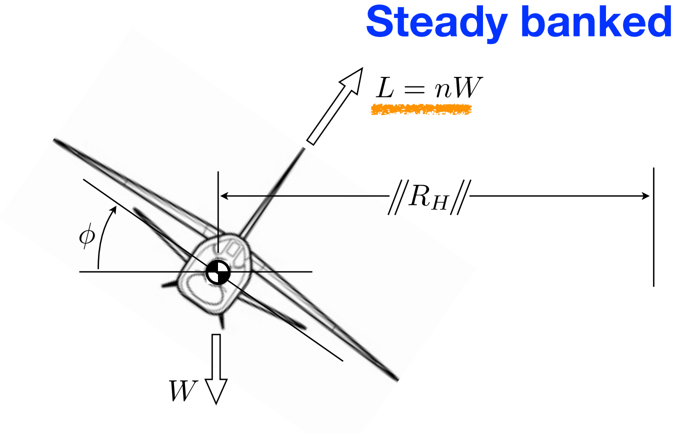
$$s_{land} \approx \frac{h_{land}}{\tan\gamma_{app}} + 1.69\frac{W/S}{\rho|\bar{a}|C_{L,\max}}$$

Note that the wing loading W/S may be much less than the maximum takeoff value, owing to fuel use.

Typical value of de-acceleration possible on a dry concrete runway is $|\bar{a}|/g = 0.3 \text{ to } 0.5$.

Note also that, unlike the case for take-off, the thrust loading T/W does not come into account here.

Steady banked horizontal turn



Horizontal equilibrium $\quad L \sin\phi = m\dfrac{V^2}{R_H} = mV\omega$

Vertical equilibrium $\quad L\cos\phi = nW\cos\phi = W$

Leading to

$$n = \frac{1}{\cos\phi} \qquad \sin\phi = \frac{\sqrt{n^2-1}}{n}$$

The lift required to turn is greater than in level flight.

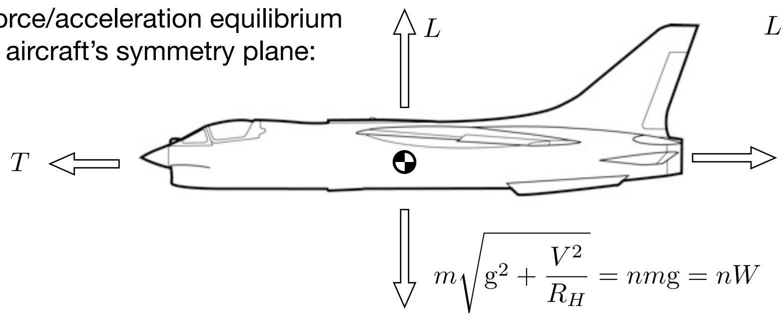
We obtain the following for rate and radius of turn:

$$\omega = \frac{\mathrm{g}\sqrt{n^2-1}}{V} \qquad R_H = \frac{V^2}{\mathrm{g}\sqrt{n^2-1}}$$

Typically we wish to either maximise the turn rate or minimise the turn radius. The first usually more important.

To consider the thrust requirement, we use force equilibrium in the tangential direction, which is $T = D$, the same as in steady level flight. However, we no longer have $L = W$, but $L = nW$, instead.

Force/acceleration equilibrium in aircraft's symmetry plane:



$$L = nW = \tfrac{1}{2}\rho V^2 S C_L \quad \text{or} \quad C_L = \frac{2}{\rho}\frac{W}{S}\frac{n}{V^2}$$

$$T = D = \frac{nW}{C_L/C_D} = nW\left(\frac{C_{D,0}}{C_L} + KC_L\right)$$

$$\frac{T}{W} = n\left(\frac{C_{D,0}}{C_L} + KC_L\right)$$

(same as steady level flight but with *nW* replacing *W*.)