

Tail surface design



Tail volume coefficients

When it comes to their stabilising effect, both the area and moment arm (from either the CG or the a.c. of the wing MAC) are important – we could trade area for moment arm and get similar effect.

The product of area and moment arm is a volume and the *tail volume coefficients* of the horizontal and vertical tails are dimensionless values of these volumes:

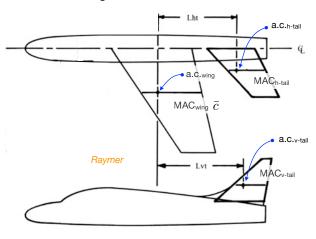
$$V_{
m ht} = rac{L_{
m ht}S_{
m ht}}{ar cS}$$

$$V_{
m vt} = rac{L_{
m vt}S_{
m vt}}{bS}$$

Note that the length scale used in the denominator is different in the two cases.

While there are design methods to establish the values $V_{\rm ht}$ and $V_{\rm vt}$ required for a particular aircraft, one finds that within a class of aircraft the values are rather similar. Here is a table of representative/indicative values.

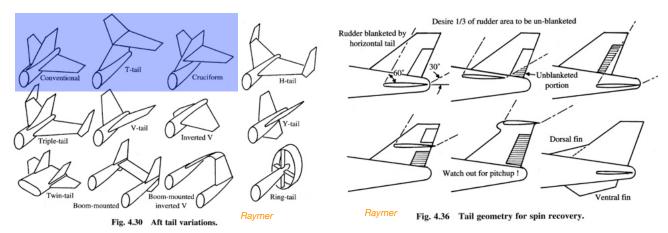
The fact that in general V_{vt} values are significantly smaller than V_{ht} values reflects the different length scales in the denominators of the definitions (although also in general $S_{vt} < S_{ht}$).



Typical values Horizontal $V_{
m ht}$ Vertical $V_{
m vt}$ Sailplane 0.50 0.02 0.50 Homebuilt 0.04 General aviation—single engine 0.70 0.04 General aviation-twin engine 0.80 0.07 Agricultural 0.50 0.04 0.90 0.08 Twin turboprop Flying boat 0.70 0.06 Ravmer Jet trainer 0.70 0.06 Jet fighter 0.40 0.07 Military cargo/bomber 1.00 0.08 Jet transport 1.00 0.09

Geometric details

- 1. A very wide variety of horizontal+vertical tail layouts exists. Unless there is a good practical reason, it's best to stay with one of the three most popular (see below).
- 2. Tail surface aspect ratios do not need to be as high as for the wing. Even though they will provide some normal force and hence, induced drag (a.k.a. trim drag), this is typically fairly small.
- 3. If the aircraft is transonic/supersonic, the tail surfaces are swept such that compressibility effects will first be felt on the wing (i.e. they typically have more sweep than the wing).
- 4. Care has to be taken in placement of the vertical tail and its control surface (rudder) so that it is effective even if the aircraft (and horizontal tail) is stalled, since the rudder is vital for stall/spin recovery.



8. Typical values of V_{ht} and V_{vt} , along with other dimensionless characteristics, vary with aircraft type according to empirical guidelines used for initial layout.

Preliminary sizing guidelines for tail surfaces

V_н Range

Alternative dimensionless sizing criteria

Aircraft Type

Personal/Utility Commuters Regional Turboprops Business Jets Jet Transports Military Fighter/Attack	.4892 .46-1.07 .83-1.47 .5199 .54-1.48 .2075	Personal/Utility Commuters Regional Turboprops Business Jets Jet Transports Military Fighter/Attack	3.5-5.0 3.5-5.0 3.5-5.0 3.5-5.0 3.5-5.0 3.0-4.0	.50-1.0 .5080 .5080 .3550 .2545 .2540	.3545 .3545 .3045 .3040 .3035 .30-1.0	.0609 .0609 .0609 .0609 .0609	
Fig. 6-10 Representative Horizontal Tail Volume Ranges		Fig. 6-17 Summary of Horizontal Tail Geometric Characteristics					Schaufele
Aircraft Type	V _v Range	Aircraft Type	AR	λ	c,/c	t/c	
Personal/Utility Commuters Regional Turboprops Business Jets Jet Transports Military Fighter/attack	.024086 .041097 .065121 .061093 .038120 .041130	Personal/Utility Commuters Regional Turboprops Business Jets Jet Transports Military Fighter/Attack	1.2-1.8 1.2-1.8 1.4-1.8 0.8-1.6 0.8 -1.8 1.2-1.6	.3050 .3080 .3070 .3080 .3080 .2540	.2545 .3545 .2545 .2535 .2540 .2035	.0609 .0609 .0609 .0609 .0810	

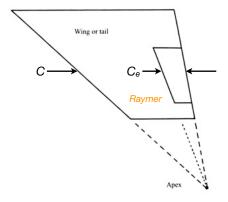
Aircraft Type

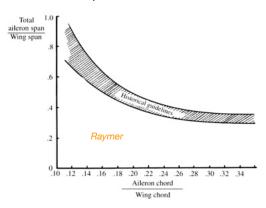
Fig. 6-16 Representative Vertical Tail Volume Ranges

- Fig. 6-18 Summary of Vertical Tail Geometric Characteristics
- 9. For an aircraft with front-mounted propeller engine, *L*_{HT} is approx. 60% of fuselage length.
- 10. For an aircraft with engines in the wings, the tail moment arm is about 50-55% of fuselage length.
- 11. For aft-mounted engines the tail moment arm is about 45-50% of fuselage length.
- 12. A sailplane has a tail moment arm of approx. 65% of fuselage length.
- 13. For an all-moving tail the volume coefficient can be reduced by approx. 10-15%.
- 14. For a T-tail the horizontal and vertical TVCs can both be reduced by approx. 5%.
- 15. For a V-tail the horizontal and vertical TVCs are estimated as above and then the V surfaces sized to provide the same total surface projected areas in each direction. The dihedral angle should be approx. 45°.
- 16. For a canard horizontal 'tail' that only provides control, the $V_{\rm ht}$ is approx. 0.1. The tail moment arm lies in the range 30–50% of total fuselage length. If the canard surface also provides lift, design as a 'two-wing' aircraft.

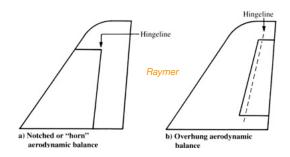
Control surfaces

- 1. Primary control surfaces are ailerons (roll), elevator (pitch) and rudder (yaw).
- 2. Control surfaces are usually tapered in chord in the same ratio as the parent.





3. Since ailerons take up part of the wingspan that we might want to use for high-lift devices (i.e. flaps), there are various compromises that allow both to be large: (a) spoilers for roll control; (b) LE slats to augment/replace flaps; (c) flaperons.



Aircraft	Elevator C_{e}	Rudder C_r/C		
Fighter/attack	r/attack 0.30 ^a			
Jet transport	0.25 ^b		0.32	
Jet trainer	0.35		0.35	
Biz jet	0.32 ^b		0.30	
GA single	0.45	Raymer	0.40	
GA twin	0.36		0.46	
Sailplane	0.43		0.40	

^aSupersonic usually all-moving only

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Tail sizing overview

- 1. Tail sizing is intimately tied to aircraft centre of gravity (CG) location, wing geometry, stability and controllability.
- 2. We usually start with wing and fuselage geometry determined, then select typical aircraft category
 - horizontal tail volume coefficient (V_{HT}) and shape,
 - · longitudinal static stability margin (SM).
- 3. If the tail placement is known approximately (e.g. from fuselage length), we can then initially size the horizontal tail area and determine the aircraft's Neutral Point (NP) and CG locations.
- 4. For further detail, we need to also know/choose the
 - · required range of CG (varies with aircraft loading, fuel use),
 - minimum allowable SM,
 - · controllability requirements in pitch.

this allows us to refine the required V_{HT} (or, if tail moment is given, S_{HT}) using a 'scissors diagram'.

- 5. NB: we have also to ensure the aircraft actually does balance at the selected CG location this may mean that we have to move aircraft component masses around (usually, by changing the location of the fuselage in relation to the wing which may then entail re-sizing the horizontal tail to achieve the required V_{HT}). Be prepared to iterate.
- 6. Vertical tail surfaces locations are by then typically known and these are usually sized and shaped according to lateral controllability (and to a lesser extent, stability) requirements.

^bOften all-moving plus elevator.

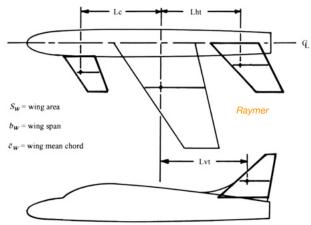
Static stability and control

- The primary purposes of tail surfaces are to provide both <u>stability</u> (i.e. the tendency to return to the same heading if disturbed) and <u>control</u> (i.e. the ability to change the heading of the aircraft by deflecting control surfaces).
- 2. Considering horizontal tail surfaces, we will mostly consider conventional layouts, i.e. tail surfaces aft of the aircraft CG location.
- We have already used some guidelines regarding typical sizings of horizontal and vertical tail surfaces when expressed in terms of tail volume coefficients.
- 4. Now we want to examine the sizing of tail surfaces in slightly more detail.
- Recall that for longitudinal stability and trim we require that, when taking moments about the aircraft CG we need both

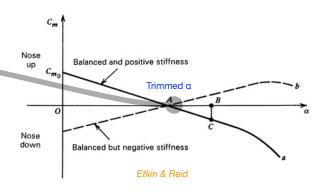
$$C_M=0$$
 and $\frac{\partial C_M}{\partial lpha}<0$

in order for the aircraft to both statically stable and trimmable at a positive C_L ,

where
$$C_M = rac{M}{qSar{c}}$$
 and $C_L = rac{L}{qS}$

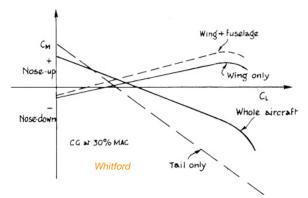


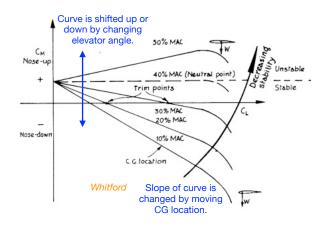
$$V_{ht} = \frac{l_{ht}S_{ht}}{\bar{c}S}, \qquad V_{vt} = \frac{l_{vt}S_{vt}}{bS}$$



Longitudinal static stability — 1

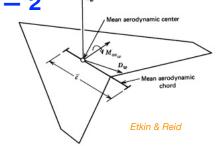
- 1. We recall that the pitching moment of the wing about the CG is typically destabilising (i.e. $\partial C_{Mw}/\partial \alpha > 0$) and the contribution of the fuselage is usually also destabilising. It is only the addition of the horizontal tail surface (which is strongly $\partial C_M/\partial \alpha < 0$) that provides overall positive pitch stiffness (stability) for the whole aircraft.
- 2. Varying the position of the aircraft CG changes the stability by altering the slope of the C_M vs α (or C_L) curve, i.e. changing $\partial C_M/\partial \alpha$.
- 3. At one particular CG position the aircraft becomes neutrally stable in pitch, i.e. $\partial C_M/\partial \alpha = 0$. This CG location is called the aircraft's <u>neutral point</u>.
- 4. The first task in designing horizontal tail surfaces is to establish the relationship between the location of the neutral point and the tail volume coefficient.
- 5. By increasing the tail volume we will move the neutral point aft.
- 6. The further the neutral point is aft of the CG, the greater the pitch stability of the aircraft, but also the more trim drag that will be incurred.
- 7. Thus if the CG is already fixed, we can size the tail surfaces to achieve any desired degree of stability, however this is not the whole story as we need also to have control authority.





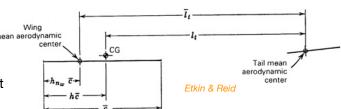
Longitudinal static stability - 2

- 8. The wing lift acts at its aerodynamic centre (typically 25% of the MAC) where a moment also acts. By definition the moment coefficient at this location is independent of α .
- 9. The distance of the wing's aerodynamic centre from the CG is expressed as a proportion of the MAC and so the moment coefficient around the CG location contributed by the wing is (approximately, since we ignore drag and the vertical location of the CG):



$$C_{Mw} = C_{Macw} + C_{Lw}(h - h_{nw})$$

10. The body of the aircraft also makes a contribution which we assume is a pure moment term acting at the CG: C_{Mb}



- 11. Recognising that the propulsion system may exert a pitching moment around the CG: C_{Mp}
- 12. Finally, the contribution of the horizontal tail (neglecting its pitching moment w.r.t. its aerodynamic centre):

$$-\frac{\bar{l}_{ht}S_{ht}}{\bar{c}S}C_{Lt} + C_{Lt}\frac{S_{ht}}{S}(h - h_{nw}) = -V_{ht}C_{Lt} + C_{Lt}\frac{S_{ht}}{S}(h - h_{nw})$$

12. Summing all the contributions:

$$C_{M,CG} = C_{Macw} + C_{Lw}(h - h_{nw}) + C_{Mb} + C_{Mp} - V_{ht}C_{Lt} + C_{Lt}\frac{S_{ht}}{S}(h - h_{nw})$$

$$\text{or} \quad C_{M,CG} = C_{Macw} + C_L(h-h_{nw}) + C_{Mb} + C_{Mp} - V_{ht}C_{Lt} \quad \text{where} \quad C_L = C_{Lw} + C_{Lt}\frac{S_{ht}}{S}$$

Longitudinal static stability - 3

13. Differentiating w.r.t. α: zero by definition

$$\frac{\partial C_{M,CG}}{\partial \alpha} = \underbrace{\frac{\partial C_{M}}{\partial \alpha}}_{\text{out}} + \frac{\partial C_{L}}{\partial \alpha} (h - h_{nw}) + \frac{\partial C_{Mb}}{\partial \alpha} + \frac{\partial C_{Mp}}{\partial \alpha} - V_{ht} \frac{\partial C_{Lt}}{\partial \alpha}$$

14. Set to zero and rearrange, where by definition $h \to h_0$, the neutral point's location:

$$h_n = h_{nw} - \frac{\partial C_{Mb}/\partial \alpha}{\partial C_L/\partial \alpha} - \frac{\partial C_{Mp}/\partial \alpha}{\partial C_L/\partial \alpha} + V_{ht} \frac{\partial C_{Lt}/\partial \alpha}{\partial C_L/\partial \alpha}$$

15. For now we will drop the propulsion derivative, as it is often, although not always, small.

$$h_n \approx h_{nw} - \frac{\partial C_{Mb}}{\partial C_L} + V_{ht} \frac{\partial C_{Lt}/\partial \alpha}{\partial C_L/\partial \alpha}$$

16. Assuming that we can assess the aerodynamics of the tail in isolation (,0) and allowing for the downwash induced by the wing as a decrement in angle of attack of the tail, ∂∈/∂α:

$$h_n \approx h_{nw} - \frac{\partial C_{Mb}}{\partial C_L} + V_{ht} \left[1 - \frac{\partial \epsilon}{\partial \alpha} \right] \eta_t \frac{\partial C_{Lt,0} / \partial \alpha}{\partial C_L / \partial \alpha}$$

where η_t allows for reduction in dynamic pressure at the tail owing to body and wing wakes, e.g. 0.9 for cruciform- and 1.0 for T-tails.

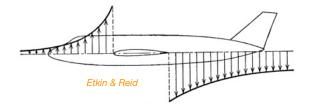
17. We can estimate the lift curve slopes of the isolated wing and tail surfaces, e.g. by

$$\frac{\partial C_L}{\partial lpha} pprox \frac{2\pi A}{2 + \sqrt{(A/\eta)^2(1 + an^2 \Lambda - M^2) + 4}}$$
 where here $\eta pprox 0.97$ typically.

We note that tail aspect ratios are usually less than for wings, hence give lower $\partial C_L/\partial \alpha$ values.

Longitudinal static stability - 4

18. We can estimate the tail downwash angle ∂ε/∂α in various ways: from wind tunnel studies, or by using linear vortex-lattice computer codes, or CFD. A simple approximation that is reasonable if the tail is well downstream of the wing is to use the value of downwash angle far behind an elliptically loaded wing:



$$\epsilon pprox rac{2C_{Lw}}{\pi A}$$
 so $rac{\partial \epsilon}{\partial lpha} pprox rac{2}{\pi A}$

19. Finally we have to deal with the fuselage term, $\partial C_{Mb}/\partial C_L$. There are various approximate approaches available, e.g. that of Multhopp. We'll use Gilruth's (NACA TR-711) quasi-empirical method, where

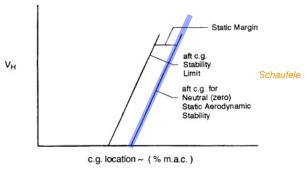
$$\frac{\partial C_{Mb}}{\partial \alpha} \approx \frac{K_f w_f^2 L_f}{S \bar{c} \, \partial C_{Lw}/\partial \alpha}$$

and w_f is the maximum width of the fuselage, L_f is the fuselage length, and K_f is an empirical factor dependent on the c/4 position of the <u>root</u> chord on the fuselage as a fraction of L_f (see also Schlichting & Truckenbrodt):

Fractional position of root chord c/4	0.1	0.2	0.3	0.4	0.5	0.6	0.7
Kf	0.115	0.172	0.344	0.487	0.688	0.888	1.146

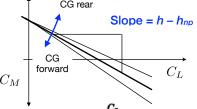
Longitudinal static stability — 5

20. At this stage we have everything we need to plot the location of the neutral point as a fraction of MAC, i.e. h_n as a function of tail volume coefficient, V_{ht} .



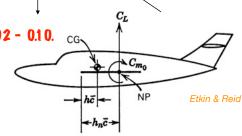
- 21. As a reminder, we typically put the aft-most position of the CG some fraction of the MAC in front of h_n . This fraction is called the <u>static margin</u>, and is typically of the order 2% to 10%, sometimes more.
- 22. By substituting the relationship for location of the neutral point back into the equation for $\partial C_{M,CG}/\partial a$

$$\frac{\partial C_{M,CG}}{\partial \alpha} = \frac{\partial C_L}{\partial \alpha} (h - h_n) \quad \text{or} \quad \frac{\partial C_{M,CG}}{\partial C_L} = h - h_n$$

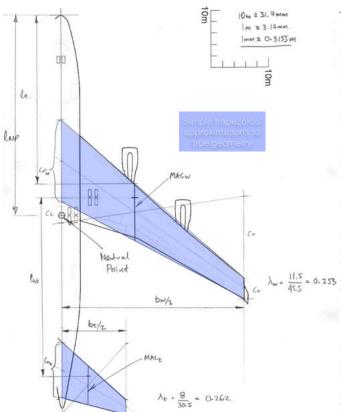


Typical values for pitch stability are (h_n-h) in the range 0.02 - 0.10. CG

23. If we find the overall pitching moment coefficient of the aircraft at zero lift, C_{M0} , we can take the total lift and moment to act at the neutral point.



For a Boeing B-747-400, estimate the distance from the nose of the aircraft to its aero dynamic neutral point. Scale lengths as required from the plan view given on the next page





JOB TITLE

NOW hus Thuw = Vht $\frac{\partial ut}{\partial a}$ $\frac{\partial u}{\partial a}$.

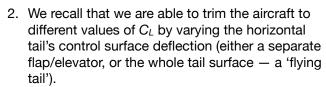
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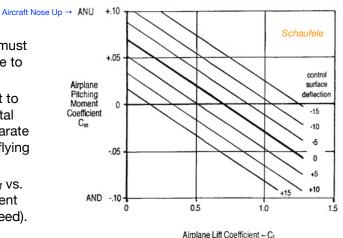
For a Boeing B-747-400, estimate the distance from the nose of the aircraft to its aeronamic neutral point. Scale lengths as required from the plan view given on the next page

Longitudinal control — 1

 The other side of tail sizing relates to controllability and control authority: the tail must be able to exert sufficient moment to be able to trim and manoeuvre the aircraft.



3. Deflecting the control surface moves the C_M vs. C_L curve either up or down to achieve different trimmed values of C_L (and hence aircraft speed).



e neutral point — we rec

- 4. Control authority is typically an issue when the CG is far forward of the neutral point we recall that the CG location can vary with load scheduling of the aircraft, or the aircraft may just be set up to be very stable.
- 5. With aft CG positions our concern is with static stability, while at forward CG locations our concern is control authority.
- 6. Examples: The tail needs sufficient authority when CG is foremost to (without itself stalling):
 - a. trim the aircraft to C_{Lmax} (stall) at altitude, aircraft clean;
 - b. trim the aircraft to C_{Lmax} in ground effect with full flap deployment (landing);
 - c. rotate the aircraft on its main wheels to take-off attitude at speed below V_{TO} (take-off);
 - d. trim and manoeuvre the aircraft at supersonic speeds (when static margin is increased).

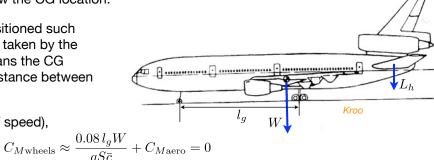
Longitudinal control - 2

- 7. A central issue is a preliminary assessment of likely CG excursion. The typical operational range of CG position varies with aircraft type.
- 8. We add this value as a proportion of MAC to the aft CG position required for static stability margin.
- 9. Then we need to size the horizontal tail (i.e. V_h) to provide the required control authority.
- 10. As a simple example, consider the case where the tail has to be able to rotate the aircraft on takeoff at a given speed (i.e. *q*) and where we know the CG location.

The landing gear is typically positioned such that at least 8% of the weight is taken by the nose gear when static. This means the CG position is approx. 8% of the distance between the nose and main gear, $0.08 \, l_g$.

At rotation speed (below takeoff speed),

Fig. 6-8 Typical c.g. Ranges

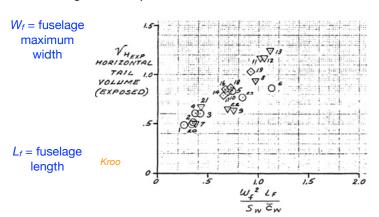


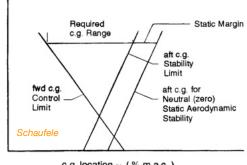
Taking the horizontal tail moment term as the largest contributor to C_{Maero} and approximating its moment arm around the wheels as I_{nt} ,

$$\frac{0.08\,l_gW}{qS\bar{c}} - \frac{qC_{L\,ht\,\mathrm{max}}S_{ht}L_{ht}}{qS\bar{c}} = 0 \qquad \text{or} \quad \frac{0.08\,l_gW}{qS\bar{c}} - C_{L\,ht\,\mathrm{max}}V_{ht} = 0 \quad \Rightarrow \quad V_{ht} \geq \frac{1}{C_{L\,ht\,\mathrm{max}}}\frac{0.08\,l_gW}{qS\bar{c}}$$

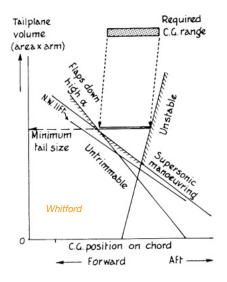
Longitudinal control — 3

- 11. (A reasonable value of $C_{L ht max}$ is approx. 1.0.)
- 12. Now we can plot the tail volume coefficient required to rotate the aircraft as a function of CG location on the same graph as used to establish V_{ht} for a given static margin. This is known as a scissors diagram, and is the primary tool for rational estimation of horizontal tail surface requirements.
- 13. We go ahead and formulate the other controllability constraints in a similar fashion and include them on the plot in order to establish the minimum value of V_{ht} required to satisfy them all at the nominated CG range.
- 14. It is advisable to compare the estimates thus obtained with available correlations, which give V_{ht} as a function of a fuselage volume parameter.





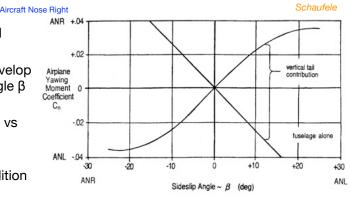
c.g. location ~ (% m.a.c.)



18

Lateral static stability & control — 1

- 1. (Aircraft CG location is taken to have been established already by longitudinal stability and control requirements and required CG range.)
- 2. Static directional stability is the tendency to develop a restoring vawing moment when a sideslip angle β is imposed.
- 3. This is assessed from a plot of yawing moment vs sideslip.
- 4. As for the longitudinal case the influence of the fuselage is typically destabilizing, while the addition of the rear vertical tail produces a stabilizing contribution, to give an overall stable result.

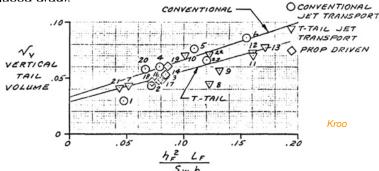


- 5. The vertical tail contribution flattens off at large angles of yaw as the vertical tail starts to stall, although this can be controlled to some extent by adding dorsal fins or using low aspect ratio vertical tail surfaces (at the expense of some extra induced drag).
- 6. Preliminary sizing (i.e. vertical tail volume coefficient) is established on a quasistatistical basis, using a measure of the destabilizing influence of the fuselage.

 h_f = fuselage maximum height

$$L_f =$$
fuselage length

$$V_{vt} = \frac{l_{vt}S_{vt}}{bS}$$



Lateral static stability & control — 2

- Just as for the horizontal tail, deflecting the control surface (rudder) produces a yawing moment which can trim the aircraft to fly at a sideslip angle (owing to yaw-roll coupling, some adverse aileron deflection is typically required also). This situation is common in the final stages of a cross-wind landing, for example.
- 2. The control authority sizing requirement that commonly determines the minimum required vertical tail surface size is the ability to fly straight with an engine-out condition at a speed close to the take-off speed, $V_{\rm mc}$. All other engines at maximum thrust.
- 3. The aircraft may be taken as on the ground or in the air, whichever is more severe.
- 4. While engine-out yawing moment may fall with airspeed, the restoring yawing moment increases with speed and tail volume coefficient V_{vt} .
- 5. $V_{\rm mc}$ is typically set with regard to the aircraft stalling speed, say $V_{\rm mc}$ < 1.2 $V_{\rm stall}$. Given this and the thrust asymmetry, one can determine $V_{\rm vt}$.



