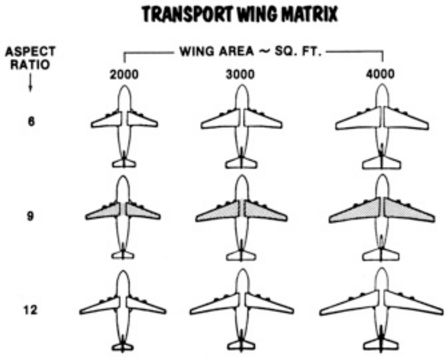
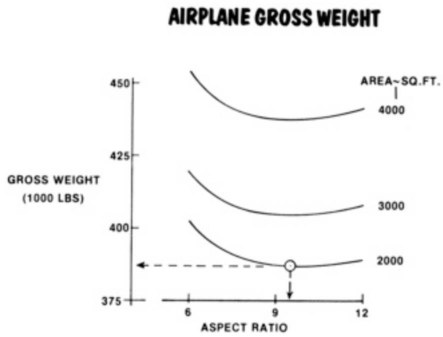
1



Design optimisation and design trades



2

Optimisation, constrained or unconstrained

Our constraint analysis methodology enables us to find workable/feasible designs in terms of W_0/S and T_0/W_0 (or P_0/W_0 for a prop-powered aircraft).

In order to pick the best/optimal solution in that space we need a scalar 'objective function' or 'cost function' which we seek to maximize (or minimize, depending on the function we pick). Contours of that function are plotted with W_0/S and T_0/W_0 as parameters.

A very typical example of a 'cost function' might be aircraft MTOW W_0 (= M_0 g). Group weight breakdowns show this to be a function of W_0/S and T_0/W_0. Recall e.g.

$$M_{\mathrm{FIXED}} + \left[C_3\left(\frac{T_0}{W_0}\right) / \left(\frac{T_0}{M_{\mathrm{ENG}}\mathrm{g}}\right) + C_4 + \frac{W_f}{W_0} - 1\right] M_0 + \overline{C}_1 \left[\frac{\mathrm{g}}{W_0/S}\right]^{0.45} M_0^{1.35} = 0$$

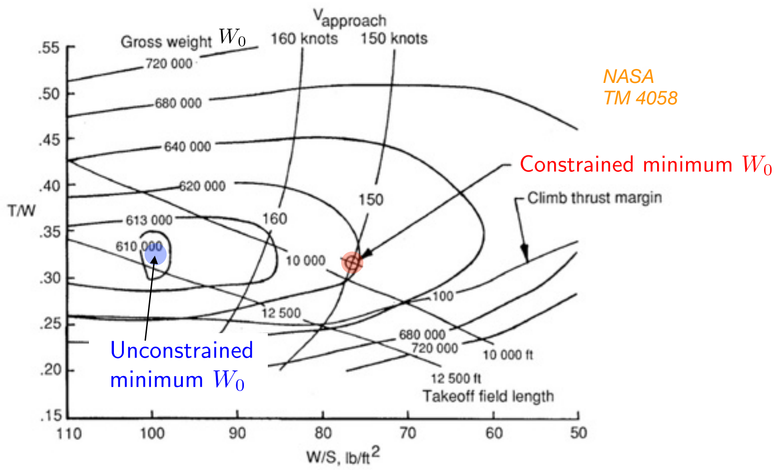
(Solve for M_0 or W_0.)

An unconstrained optimisation problem to minimise W_0 would find the (W_0/S, T_0/W_0) coordinate that gives minimum W_0.

The constraint lines on the plot turn the problem into one of constrained optimisation: we have to find minimum W_0 *subject to performance constraints.*

Either only a single constraint line or more typically the intersection of two constraint lines produces the constrained optimum. (We have to check which intersection is the governing one.)

W_f/W_0 may also be a function of W_0/S.



(Note reversed sense of W_0/S scale in this example.)

Choice of cost function (a.k.a. figure of merit)

We will often use W_0 as the cost function, but there are many possible choices.

Below are a number of other possibilities cited by Nicolai & Carichner:

- **Takeoff weight.** Indicates the general vehicle size and hence cost and energy requirements
- **Cost.** The total life cycle cost (LCC) over a fixed period such as 10 years; tradeoff between RDT&E, acquisition, and O&M costs
- **Energy.** Total fuel required for mission
- **System effectiveness.** Some parameter that combines performance, cost, and/or energy, such as the following:
  - Return on investment (ROI)
  - Bombs on target per hour per dollar
  - Kill ratio per aircraft dollar
  - Survivability
  - Transport direct operating cost (DOC)
  - Energy effectiveness parameter

These could alternatively be used as the ordinate for the carpet plot OR appear as contour lines on a constraint plot.

Adding a cost function to the constraint plot

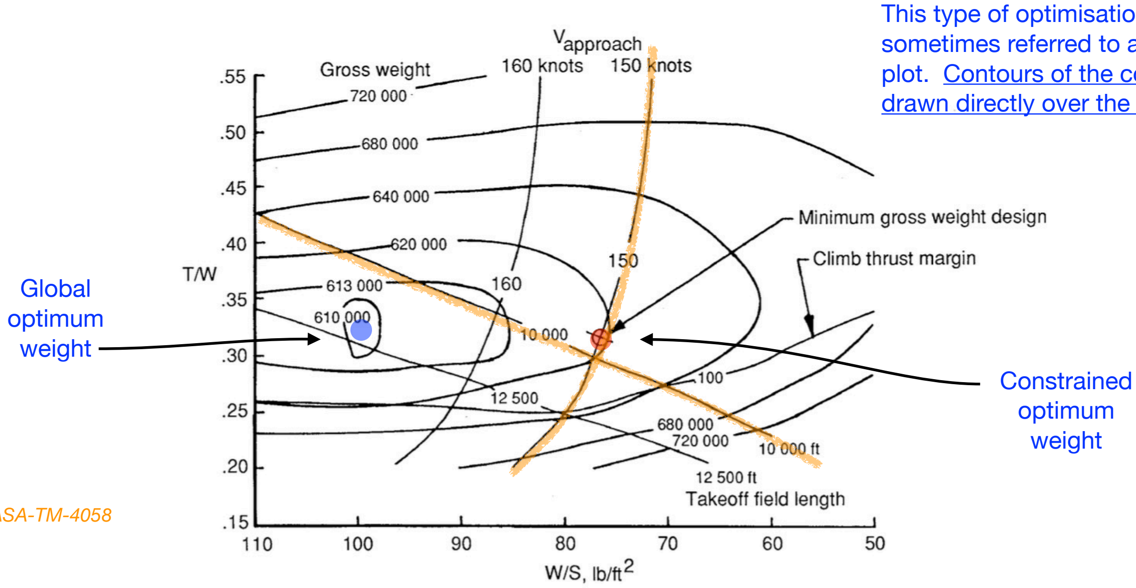
So far our mission analysis, group weight estimates, and constraint analysis have been only weakly coupled.

Also, optimal performance for the main mission task (e.g. maximising range) was not directly incorporated with matching for constraints.

Finally, there is no easy way of finding an optimum w.r.t. some cost function (e.g. minimizing W_0).

One way around the last of these issues is to add contour lines of cost function (e.g. W_0) to the constraint plot. This is possible since we can now estimate aircraft weight for every T_0/W_0 and W_0/S pair.

Every (T_0/W_0, W_0/S) pair requires a separate mission analysis and group weight computation.

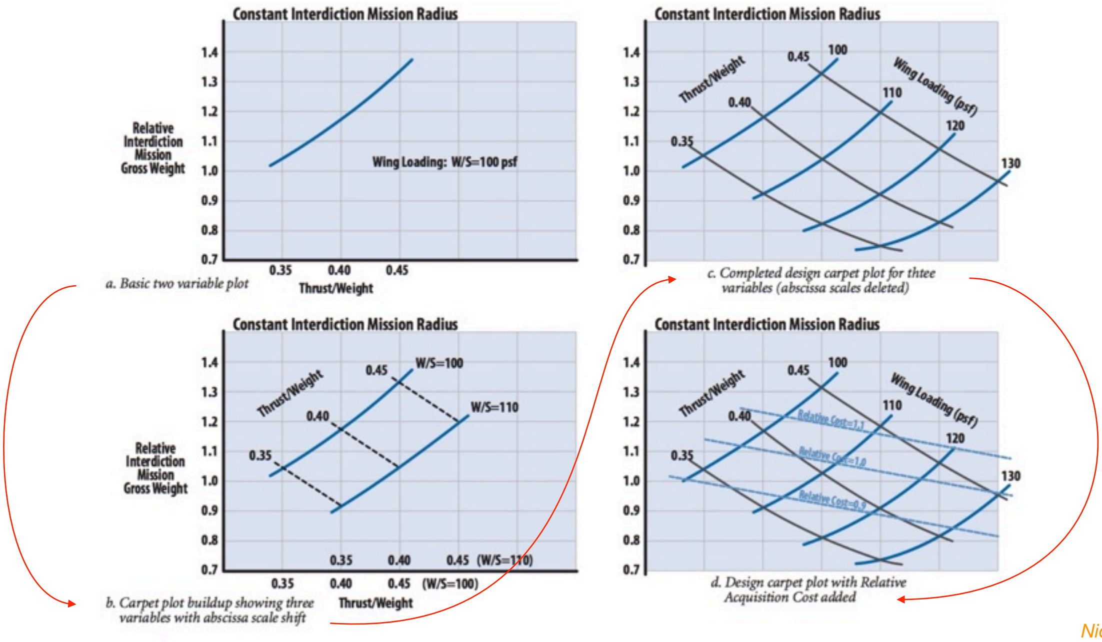


This type of optimisation chart is sometimes referred to as a 'knothole' plot. Contours of the cost function are drawn directly over the constraint plot.

NASA-TM-4058

Carpet plots – an alternative

Carpet plots are another commonly used way of displaying 3, 4, or more, variables on a 2D plot. (In fact what we'll look at initially are of 3-variable type, sometimes called "Cheater Plots".)
Essentially they show the same information as a contour plot but in a different way.



Nicolai & Carichner

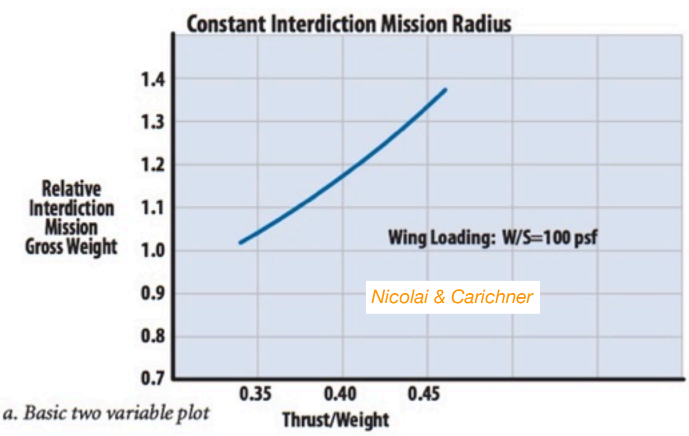
Figure 25.3 Example of design carpet plot buildup for a Navy multimission fighter.

This example is convenient because T/W and W/S are used to generate the 'carpet', and W_0 is the cost function.

Carpet plots

First we plot (the cost function) W_0 as a function of T_0/W_0 for a constant wing loading W_0/S.

(The discourse below is for a jet aircraft. If considering a propeller aircraft, substitute P_0/W_0 for T_0/W_0, and adjust equations accordingly.)



Choose a fixed value of W_0/S. For supplied cruise conditions this implies C_L.

$$C_L = \frac{\beta}{q}\frac{W_0}{S}$$

For steady level flight, we have

$$\frac{T_0}{W_0} = \frac{\beta}{\alpha}\frac{1}{C_L/C_D}$$

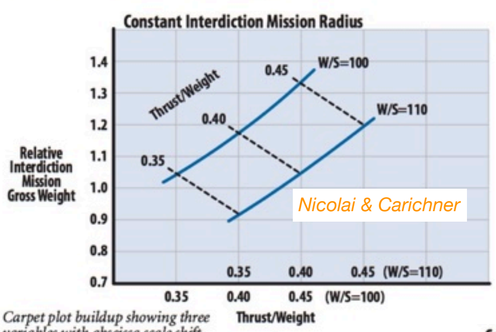
where $C_L/C_D = C_L/C_D(C_L)$ uses the aircraft drag polar model for C_D.

This means we have T_0/W_0 for a given W_0/S.

Using the aircraft's group weight correlations and mission analysis for energy/fuel use, we estimate W_0 for different T_0/W_0 with given W_0/S and can plot one line of a carpet plot.

Note that in the fuel use part of the weight estimation, we use the value of C_L (and C_D) determined above.

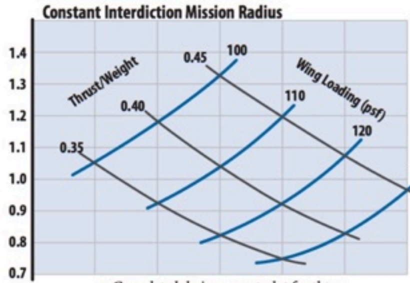
Carpet plots



b. Carpet plot buildup showing three variables with abscissa scale shift

We repeat the process for another value of W_0/S, and plot another line. Note the abscissa for plotting this line is offset. Join the shared T_0/W_0 values together and label those new lines.

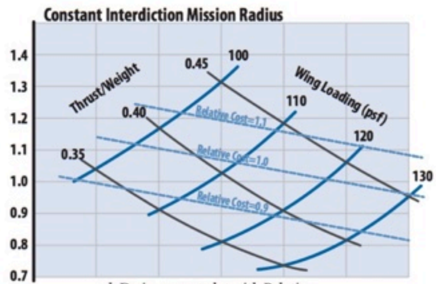
We complete the carpet plot using further values of W_0/S. Then the abscissa can be deleted.



c. Completed design carpet plot for three variables (abscissa scales deleted)

There is a Matlab toolbox that claims to be able to do carpet plots. Not verified to work!

Now we could plot further curves which are functions of T_0/W_0, W_0/S, and, if required, W_0.

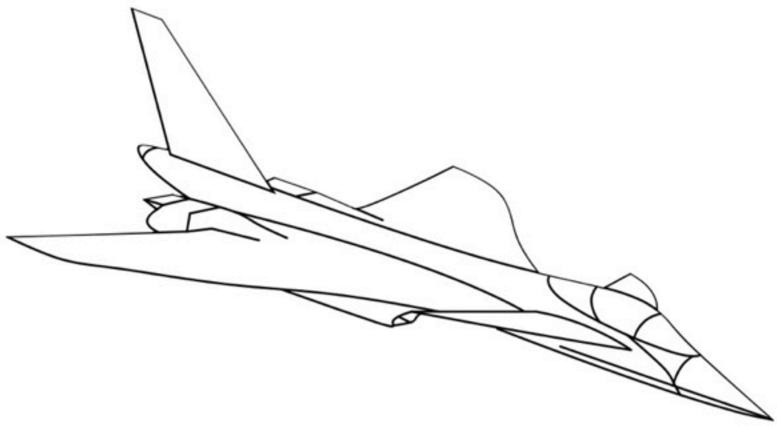


d. Design carpet plot with Relative Acquisition Cost added

Because the carpet plot we've generated here is for T_0/W_0 and W_0/S, we can draw the performance constraint curves on it.

Carpet plot example

- **Supersonic fighter**
  - **Examine wing loading and thrust loading**
  - **Understand W/S and T/W sensitivity and impact of constraints**
    - **Weight to meet mission requirements**
    - **Effect of M 0.9 sustained manoeuvre reqmnt @ 30 kft**
    - **Acceleration M 0.9 to 1.6 @ 30 kft**
    - **Field performance**



Adapted from Mason, NASA-CR-3763

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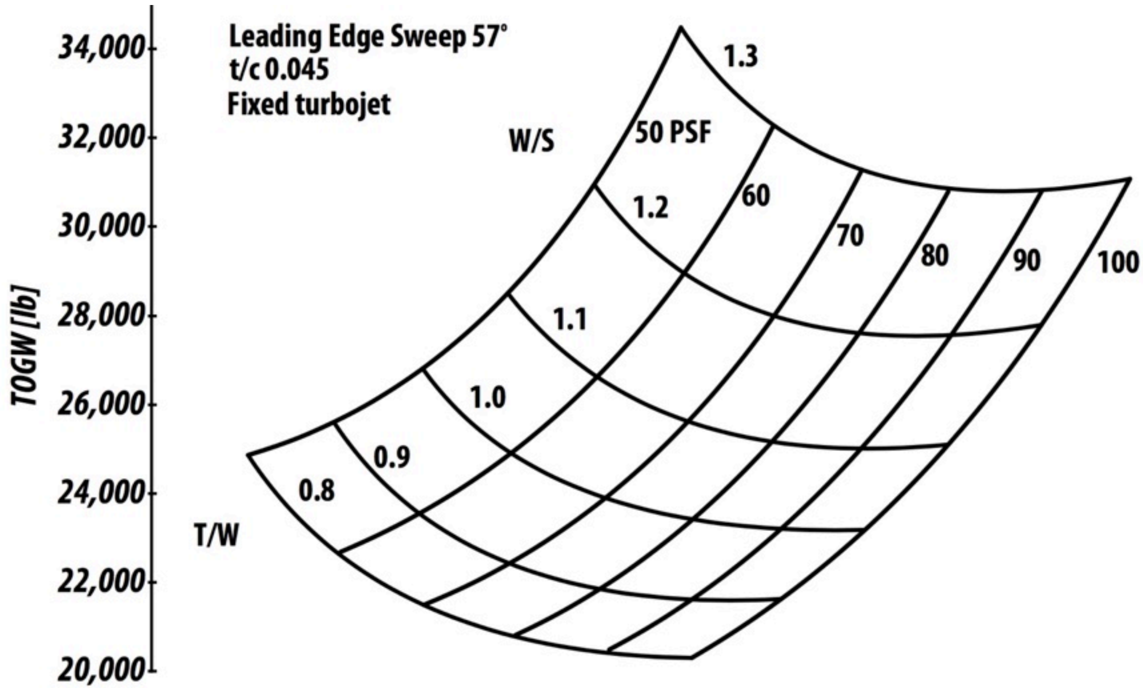
Carpet plot example



Adapted from Mason, NASA-CR-3763

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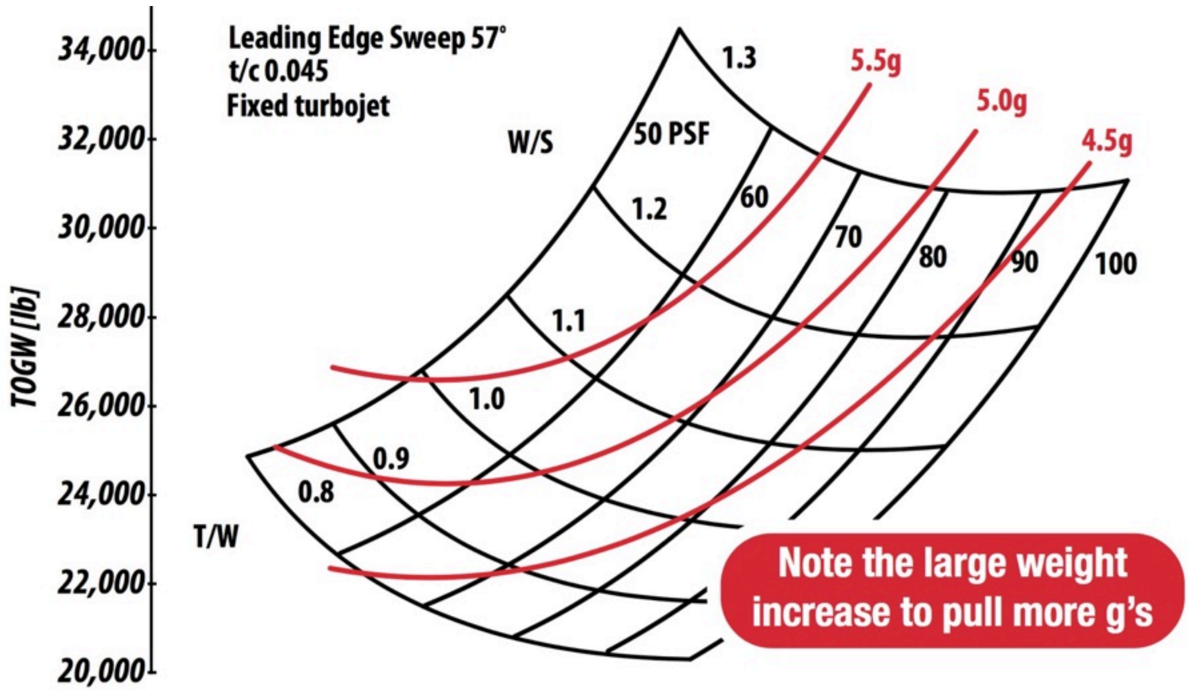
Carpet plot example



Adapted from Mason, NASA-CR-3763

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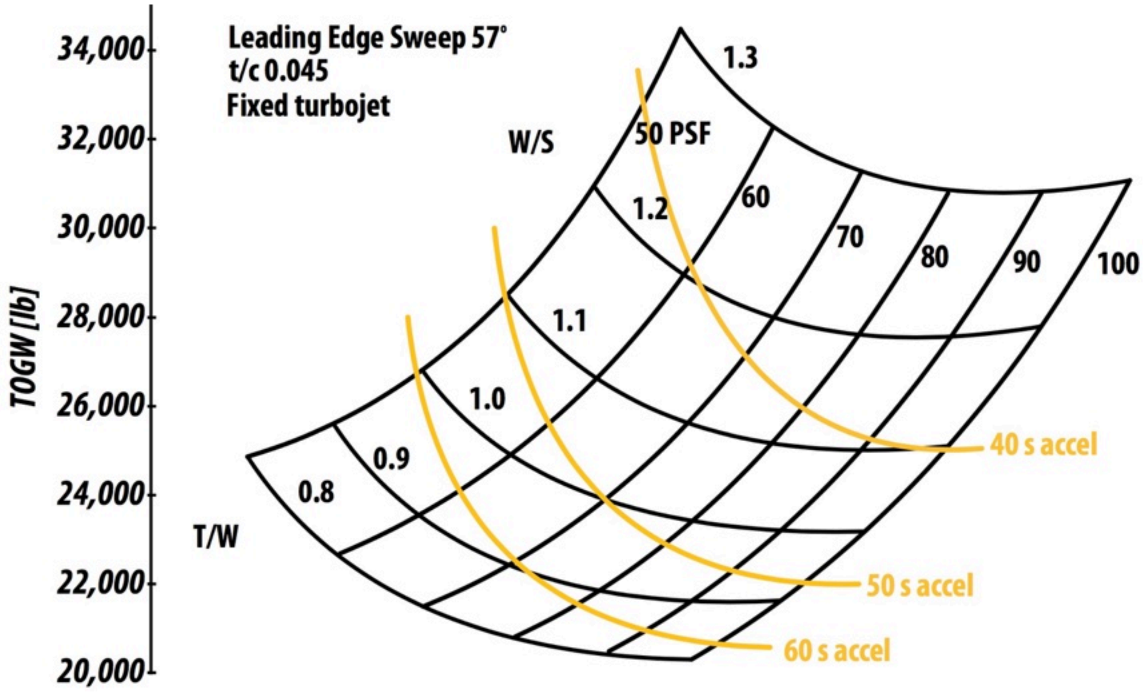
Carpet plot example



Adapted from Mason, NASA-CR-3763

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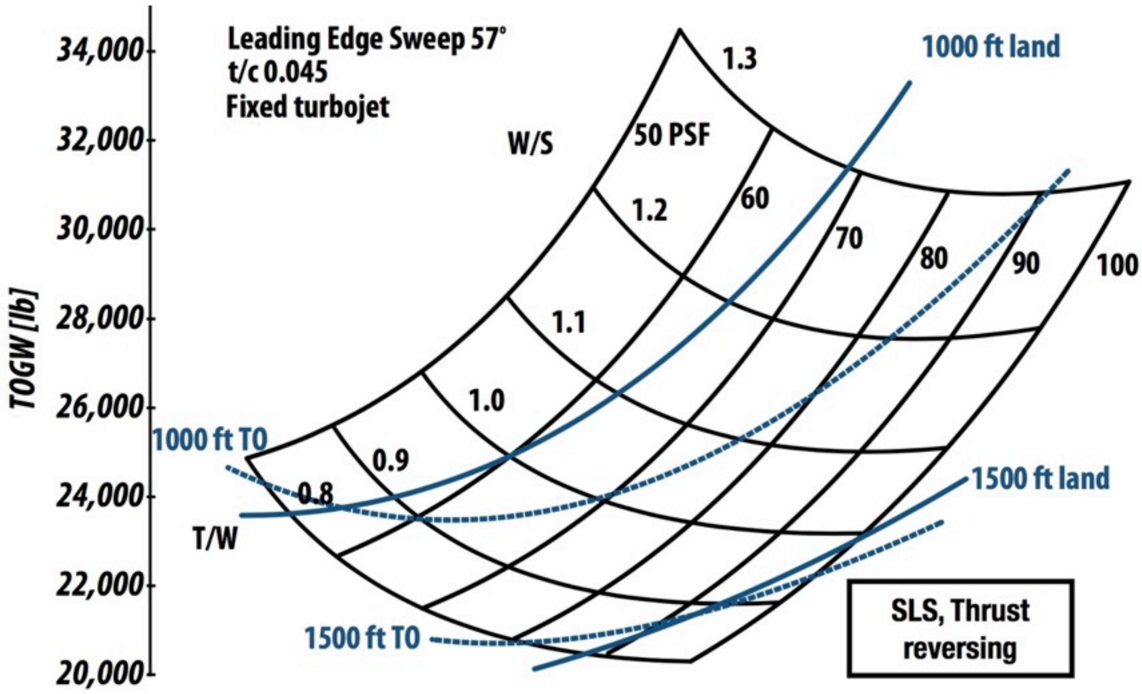
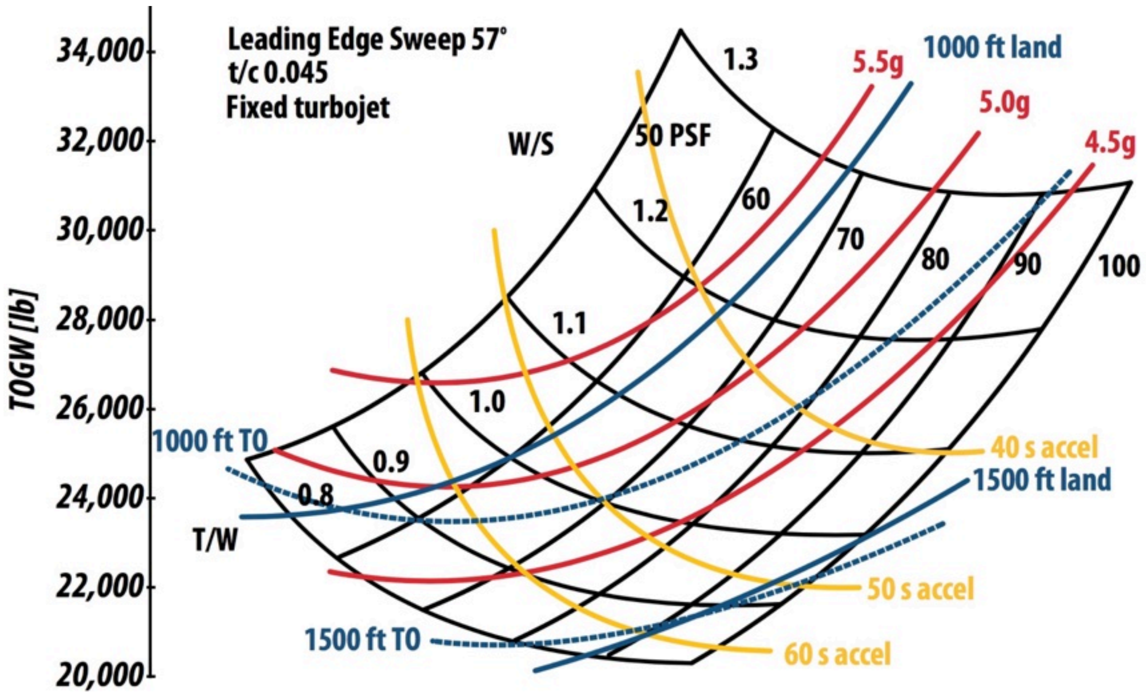
Carpet plot example



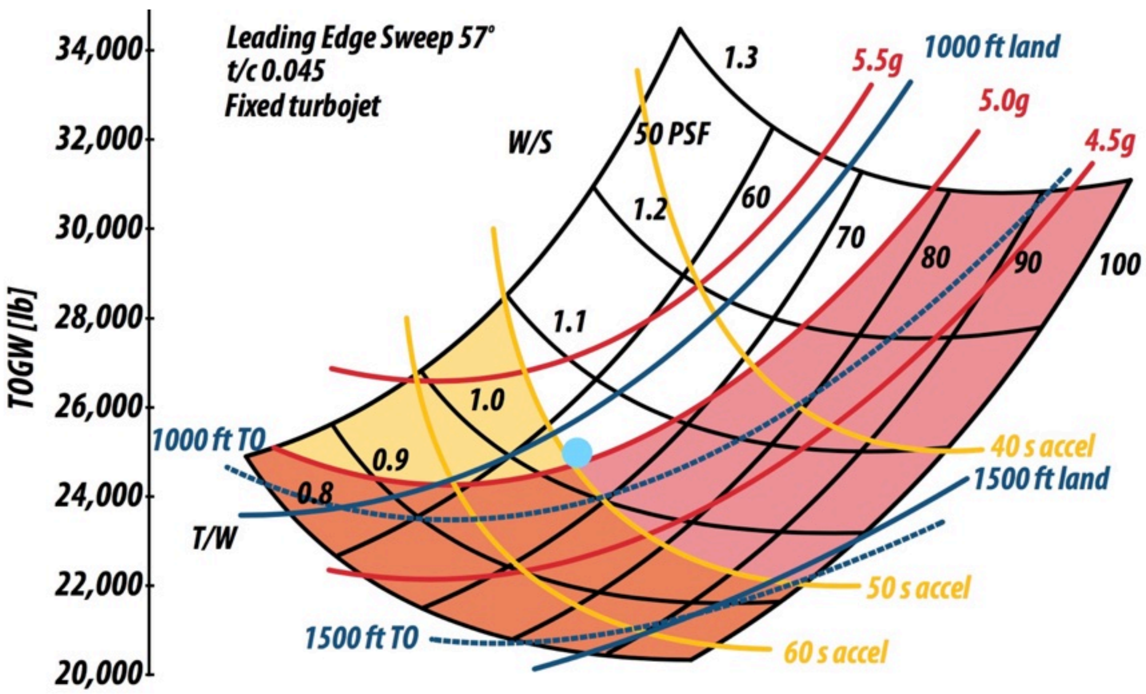
Adapted from Mason, NASA-CR-3763

Carpet plot example



Adapted from Mason, NASA-CR-3763

Carpet plot example



Adapted from Mason, NASA-CR-3763

Parametric variation – 1

We can use either contour or carpet plots to examine the effect of design parameters on the (constrained) optima of our cost function (below, the cost function is W_0, a.k.a. TOGW, and the parameter varied is wing aspect ratio A). This enables us to see how varying a single design parameter influences the constrained optimum.

Nicolai & Carichner

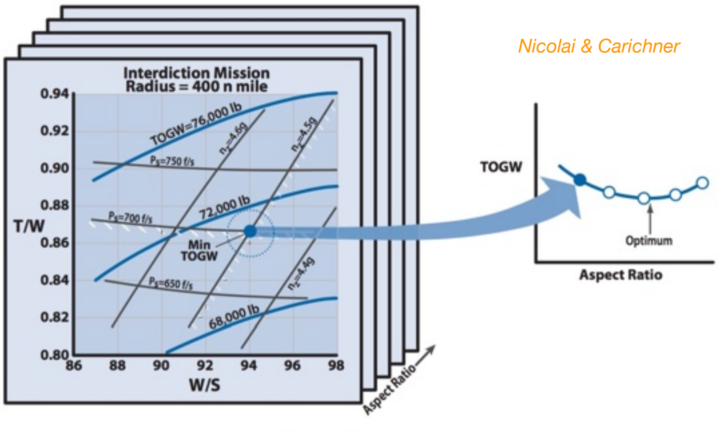
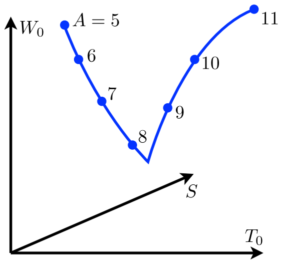


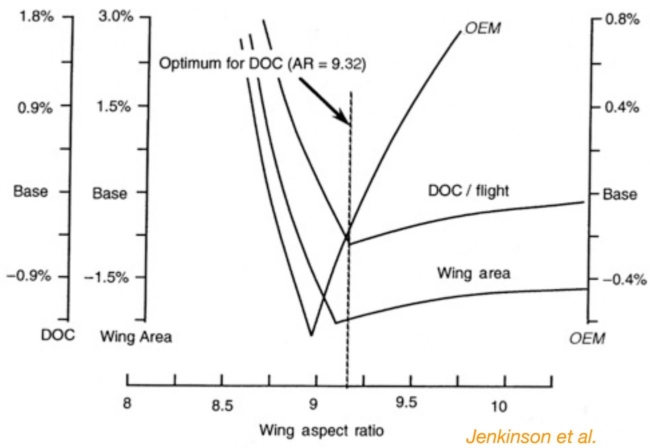
Figure 25.2 Parametric tradeoff showing a three-variable example of wing loading, thrust-to-weight ratio, and aspect ratio.



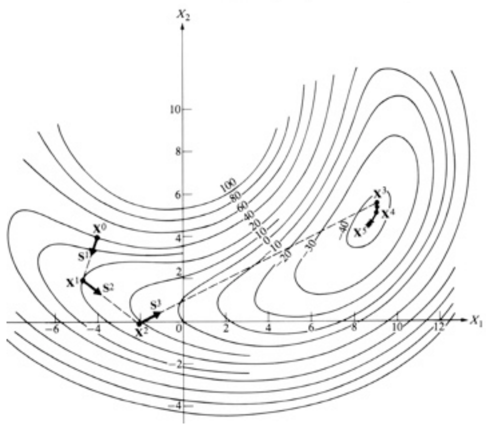
Note that the plot of optimum W_0 as a function of A may have kinks or slope discontinuities; this would occur if changing A meant that the constrained optimum moved to the intersection of a new pair of constraint curves.

Parametric variation — 2

2. Note again that by varying each parameter we end up with a complete new design that satisfies all the requirements and for which we can carry out a complete analysis for any figure of merit we choose.
3. As we noted previously any outcome point in the design triple space of (W_0, T_0, S) tends to lie at the intersection of a pair of design constraints/ inequalities. Thus as we vary a design parameter we may find slope discontinuities in the figure of merit. This is caused by our constrained design point moving from the intersection of one pair of design constraints to another (but retaining one of the original pair of constraints).
4. Of course, as we indicated there are a number of formal general/mathematical/computational approaches to optimisation. These have made significant progress in optimisation of specific parts and also of aerodynamic shape.



Jenkinson et al.



A gradient-following/steepest descent algorithm for unconstrained optimisation.

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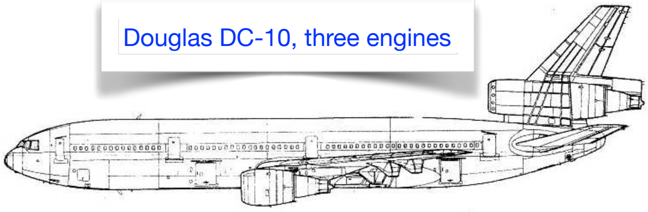
Design trades

1. We can vary simple design layout decisions (e.g. number of engines) with most other variables fixed and see how that may influence weight, cost, or a key performance parameter (for the DC-10, this was takeoff distance, although the direct operating cost increased). Other design trades are produced by parametric geometric variations.



Douglas DC-10, three engines

MODEL D966
GENERAL CHARACTERISTICS
TWO ENGINE DESIGN

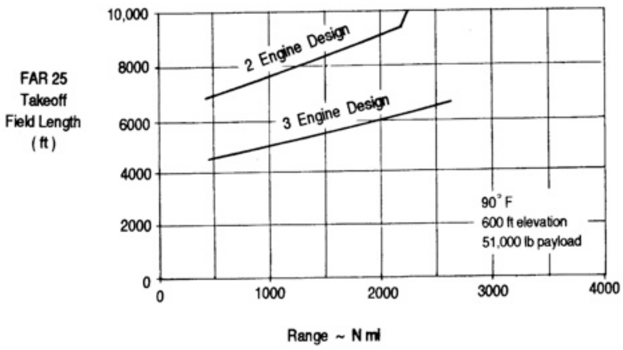
WING AREA	3,530 SQ FT.	TAKEOFF WEIGHT (LB)	318,000
ASPECT RATIO	8.8	O.W.E. (LB)	202,400
SWEEP	30.5°	RANGE (N.MI)	1,850
		TAKEOFF DISTANCE (FT)	9,000
		CRUISE MACH NO.	.80
		STALL SPEED (KN)	95
MIXED CLASS 230 PASS. + 5,000 CARGO (51,000 LB PAYLOAD)		ENGINE THRUST (LB/ENG)	44,000
		PASSENGER CAPACITY 250 ALL COACH @ 36 IN PITCH	

MODEL D967
GENERAL CHARACTERISTICS
THREE ENGINE DESIGN

WING AREA	3,490 SQ FT.	TAKEOFF WEIGHT (LB)	325,000
ASPECT RATIO	7.0	O.W.E. (LB)	201,400
SWEEP	30.5°	RANGE (N.MI)	1,850
		TAKEOFF DISTANCE (FT)	5,450
		CRUISE MACH NO.	80
		STALL SPEED (KN)	96
MIXED CLASS 230 PASS. + 5,000 CARGO (51,000 LB PAYLOAD)		ENGINE THRUST (LB/ENG)	32,000
		PASSENGER CAPACITY 250 ALL COACH @ 36 IN PITCH	

2 VS 3 ENGINE COMPARISON
CRUISE SPEED MACH .80

		NUMBER OF ENGINES 2	vs 3
MODEL NO.		D966	D967
ENGINE THRUST	(LB/ENG)	44,000	32,000
TAKEOFF WT	(LB)	318,000	325,000
OPERATING WT EMPTY	(LB)	202,400	201,000
WING AREA	(SQ FT)	3,530	3,490
ASPECT RATIO		8.8	7.0
TAKEOFF FIELD LENGTH 600 FT ALT 90°F	(FT)	8,000	5,450
TAKEOFF WT FOR LGA—ORD	(LB)	287,000	291,500
DOC 250 PASS.	(¢/SEAT ST MI)	.746	.787



Schaufele

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Requirement trades — 1

1. The idea in requirement trades (a.k.a. mission trades) is to see how sensitive the cost function is with respect to variation in stated performance requirements. This information can be used to make rational choices about the impact of relaxing/tightening these requirements (which may be somewhat arbitrary).

MODEL D967
DESIGN RANGE TRADE STUDY

		RANGE (N MI) 1850	vs 2500
CRUISE MACH NUMBER		0.85	0.83 to 0.85
TAKEOFF WT	(LB)	335,000	358,500
OPERATING WT EMPTY	(LB)	210,200	211,800
TAKEOFF FIELD LENGTH 90°F, 600 FT ALT MAX T O G W	(FT)	5,500	6,600
TAKEOFF: FIELD LENGTH	(FT)	4,350	4,500
90°F, LGA—ORD WEIGHT	(LB)		
TAKEOFF: WT FOR LGA—ORD	(LB)	301,000	303,000
DOC (RANGE 1850 N MI). 250 PASS ALL - COACH	(¢/SEAT ST MI)	.781	.784

CONSTANT: ASPECT RATIO 7.0; STALL SPEED 96 KNOTS; PAYLOAD 51,000 LB; ENGINE THRUST S.L.S.: 3 X 34,500 LB

MODEL D967
STALL SPEED TRADE STUDY

		KNOTS ($V_{S_{1g}}$) 96	vs 100
ENGINE THRUST S.L.S.	(LB)	3 X 34,500	3 X 34,000
WING AREA	(SQ FT)	3,790	3,400
TAKEOFF WT	(LB)	335,000	327,000
OPERATING WT EMPTY	(LB)	210,200	203,300
TAKEOFF FIELD LENGTH 90°F, 600 FT ALT • MAX TOGW	(FT)	5,500	5,900
TAKEOFF: FIELD LENGTH	(FT)	4,350	4,750
90°F, LGA—ORD WEIGHT	(LB)	301,000	295,000
DOC 250 PASS - ALL COACH	(¢/SEAT ST MI)	.781	.762

CONSTANT: CRUISE MACH .85; ASPECT RATIO 7.0; WIND SWEEP 37.5°; DESIGN RANGE 1,850 N MI; PAYLOAD 51,000 LB

Schaufele

MODEL D967
CRUISE SPEED TRADE STUDY

		MACH .80	vs .85
ENGINE THRUST S.L.S.	(LB)	3 X 32,000	3 X 34,500
WING AREA	(SQ FT)	3,490	3,790
WING SWEEP	(°)	30.5	37.5
TAKEOFF WT	(LB)	325,000	335,000
OPERATING WT EMPTY	(LB)	201,000	210,200
TAKEOFF FIELD LENGTH 90°F, 600 FT ALT • MAX T O G W	(FT)	5,500	6,600
TAKEOFF: FIELD LENGTH LOA—ORD/FIELD LENGTH (FIELD LENGTH - FT/T O C W - LD)	(FT)	4,200 FT/291,500 LB	4350 FT/301,000 LB
DOC 250 PASS. ALL COACH	(¢/SEAT ST MI)	.787	.781

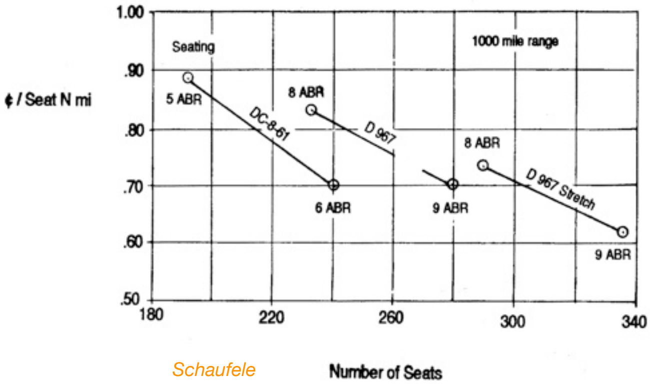
CONSTANT: ASPECT RATIO 7.0; STALL SPEED 96 KNOT; DESIGN RANGE 1,050 N MI; PAYLOAD 51,000 LB; INITIAL CRUISE ALTITUDE 35,000 FT

DC-10 TYPE
TRADE STUDY SUMMARY
RANGE 1850 N MI

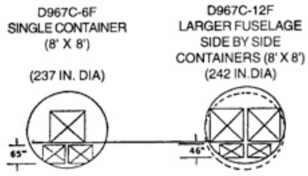
FLEXIBILITY OF 3 ENGINE	vs	2 ENGINE
	COSTS	+7000 LB TAKEOFF GROSS WEIGHT (SAVES 1400 LB OPERATING WEIGHT EMPTY) .04¢/SEAT ST MILE DIRECT OPERATING COST
TRANSCONTINENTAL RANGE CAPABILITY	COSTS	2000 LB TAKEOFF WEIGHT .01¢/SEAT ST MILE DIRECT OPERATING COST
CRUISE MACH NO. INCREASE .80 TO .85	COSTS	10,000 LB TAKEOFF WEIGHT NO CHANGE IN DIRECT OPERATING COST
STALL SPEED INCREASE FROM 96 TO 100 KNOTS	COSTS	8000 LB TAKEOFF WEIGHT .02¢/SEAT ST MILE DIRECT OPERATING COST

Requirement trades — 2

2. Aircraft commonly evolve by stretching the fuselage, together with adopting more powerful engines as these become available with time. Decisions about these evolutions may be initiated while still in the preliminary design stage. For example, wing area may be left larger than strictly optimal in order to more readily accommodate future fuselage stretch (and more fuel).
3. In a similar vein, fuselage cross-section profile and sizing may be examined for future cargo transport use options.

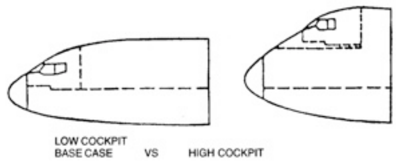


FUSELAGE CROSS SECTION COMPARISONS



		D967C-6F	D967C-12F
VOLUME			
UPPER DECK CONTAINERS		6,950	11,000
LOWER COMP CONTAINERS + BULK		3,590	2,320
		TOTAL 10,540 CU FT	TOTAL 13,320 CU FT
▲ DEAD WEIGHT	(LB)	0	+700
▲ DRAG	(SQ FT)	0	.62
▲ TAKEOFF G.W. WT	(LB)	0	2500
▲ OWE	(LB)	0	1640
▲ ENGINE SIZE	(LB/ENG)	0	500
▲ DOC (ALL COACH)	¢/SEAT ST MI)	0	.005

DC-10
LOW vs HIGH COCKPIT



	LOW COCKPIT	VS HIGH COCKPIT
△ FUSELAGE LENGTH (FT)	0	-6
△ DEAD WEIGHT (LB)	0	-800
△ DRAG (SQ. FT.)	0	+0.68
△ TAKEOFF G.W. (LB)	0	-70
△ O.W.E. (LB)	0	-670
△ WING AREA (SQ. FT.)	0	-7
△ ENGINE S.L.S.T. (LB)	0	+280
△ DOC (ALL COACH) (¢/SEAT ST. MI.)	0	-.0005

Technology trades

Technology trades examine the sensitivity of the cost function (e.g TOGW/MTOM) to single-parameter variations in design parameters/technological features when simultaneously all performance requirements are met.

This aids assessment of both (a) what technological improvements have maximum impact on the cost function and conversely (b) what the potential risks are in some technology not performing as expected.

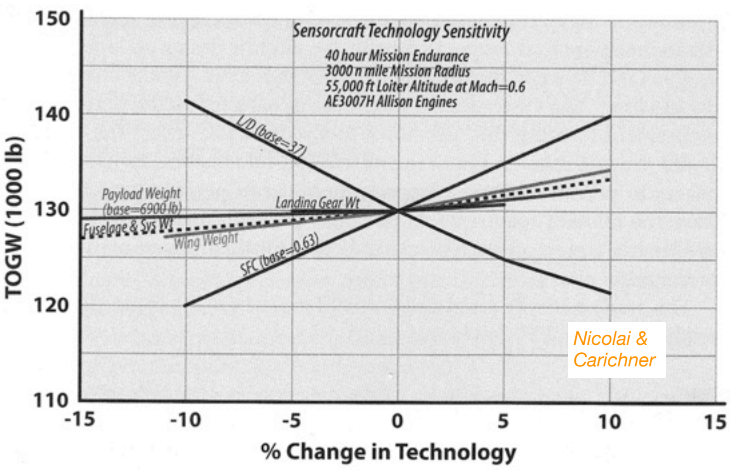


Figure 25.7 ISR aircraft technology trade results.