Aerodynamics of airfoils

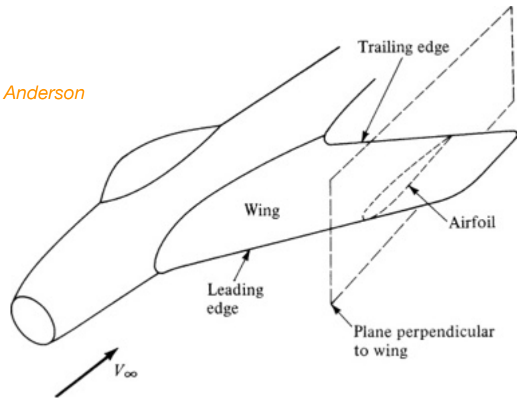
Torenbeek & Wittenberg Ch 4

Anderson Ch 5

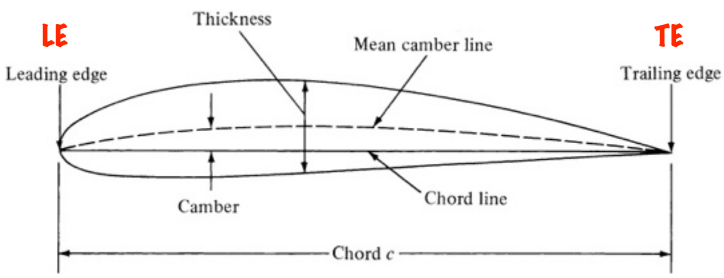
Airfoil nomenclature

Flows past airfoils are the principal means by which lift is created – and lift keeps aircraft in the air.

Anderson



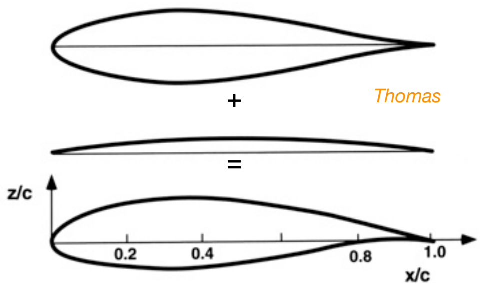
An airfoil is a 2D slice through a 3D wing on a plane parallel to the aircraft plane of symmetry.



The leading descriptors of an airfoil are its chord c, its maximum thickness and maximum camber.

The front-most point on an airfoil is called the leading edge (LE), while the rearmost point is called the trailing edge (TE). The line that runs between these is the chord line, length c. *This chord line is the typical (but not the only) reference line for an airfoil.*

Any airfoil shape can be decomposed into a symmetrical thickness distribution, and a mean camber line about which this thickness is equally distributed top and bottom.



Thomas

Airfoils are not required to have any camber in order to produce lift, but for practical reasons (e.g. strength) typically have finite thickness

In general, the key requirements for an efficient subsonic airfoil are that it have a rounded nose, and a sharp trailing edge shape.

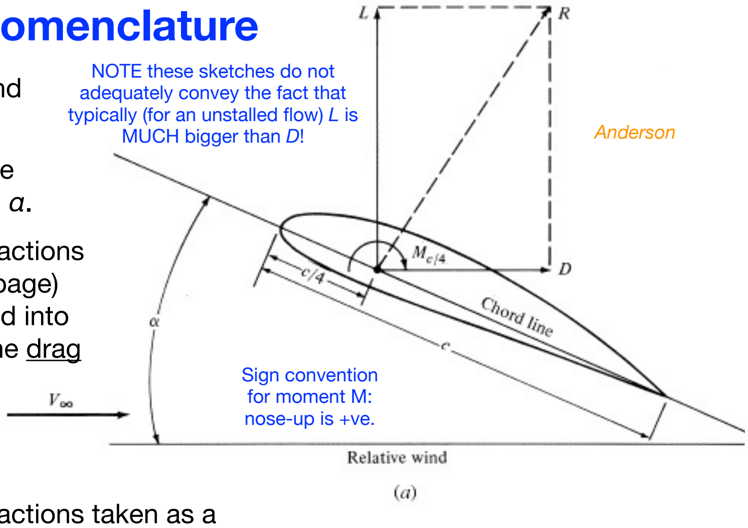
Airfoil nomenclature

The free-stream flow speed far upstream is V_∞ and its direction is called the relative wind.

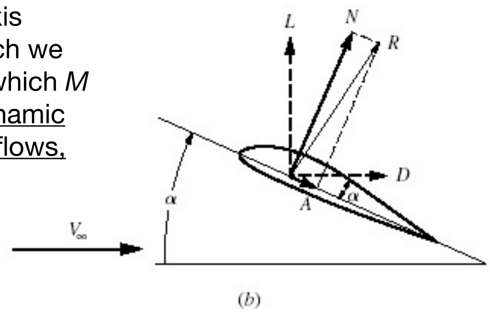
The angle airfoil chord line makes with the relative wind is called the angle of attack or AoA, symbol α.

The integrated effects of pressure and viscous tractions create an overall force (per unit width out of the page) which here we call R, conventionally decomposed into the lift force L, normal to the relative wind, and the drag force D, which is parallel to the relative wind.

The forces per unit width (R, L, D) are all functions of α.



The integrated effects of pressure and viscous tractions taken as a vector cross product with radius relative to some reference axis create an overall moment (per unit width out of the page) which we call M. It is usual to take this reference axis at a location for which M is essentially independent of α: this point is called the aerodynamic centre for the airfoil, and we have M_{ac} = const. For subsonic flows, this aerodynamic centre is very close to the $c/4$ location.



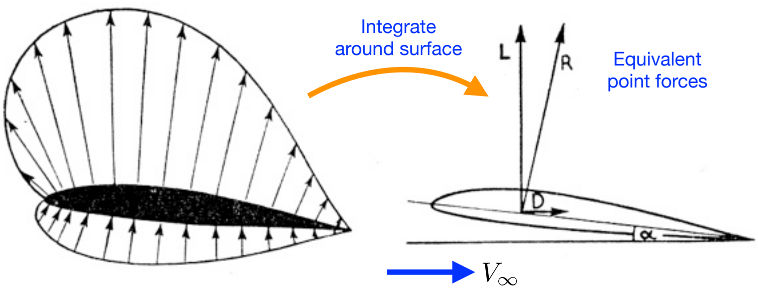
We could also decompose the reaction force R into components directed normal to and aligned axially with the chord line, which we call N and A, respectively.

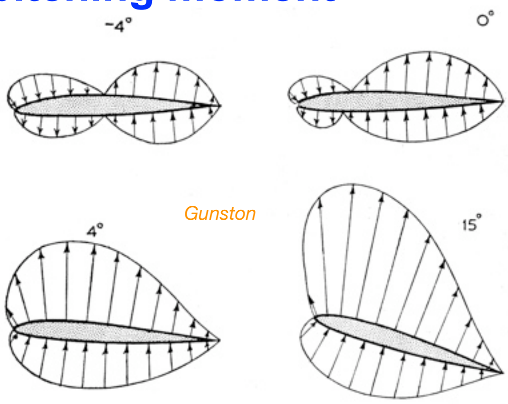
From geometry:

$$
\begin{aligned}
L &= N\cos\alpha - A\sin\alpha \\
D &= N\sin\alpha + A\cos\alpha
\end{aligned}
$$

L and D is the most common decomposition of R used for airfoils.

Pressure distributions, lift and pitching moment





The largest contributions to lift force and moment on an airfoil come from the pressure distribution around it.

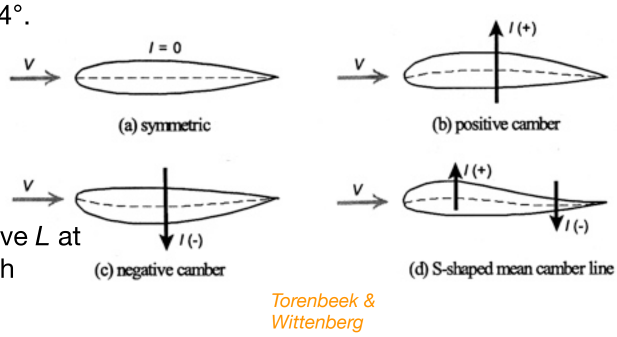
One way of visualizing the pressure distribution is to draw a series of vectors around the airfoil, locally normal to the surface, that point outward if the pressure is lower than the far-field pressure (i.e. for relative suction) and inwards if the pressure is higher than the far-field. Distributions change with Angle of Attack (AoA) α.

For airfoils with positive camber, there is nose-down pitching moment (typically measured about the 1/4 chord point, $M_{c/4}$) for all angles of attack. This is most easily seen from the pressure distribution when there is no lift, which in the figures above occurs near $\alpha = -4°$.

For a symmetrical airfoil (one with no camber) $M_{c/4} = 0$ (at all α) and zero lift is produced at (and only at) $\alpha = 0$.

Positive (negative) camber generates upward (downward) force at $\alpha = 0$, with $M_{c/4}$ negative (positive) at all α.

Reflexed (S-shaped) camber lines may have either +ve or -ve L at $\alpha = 0$. Used when nose-up pitching moment is needed with positive L.



Lift, drag and moment coefficients for airfoils

It is standard to present airfoil lift, drag forces and moments per unit width, and also in dimensionless form. Where a length is needed for the non-dimensionalisation, we use the chord length *c*.

Lift coefficient $C_l = \dfrac{L}{\frac{1}{2}\rho V_\infty^2 c}$

Drag coefficient $C_d = \dfrac{D}{\frac{1}{2}\rho V_\infty^2 c}$

Moment coefficient $C_m = \dfrac{M}{\frac{1}{2}\rho V_\infty^2 c^2}$

$[C_l]$ = [force per unit length / pressure×length] = [1]

In general, these coefficients are functions of angle of attack α, chord Reynolds number $Re = \rho V_\infty c/\mu$, and the Mach number (V_∞ / speed of sound), as discussed by Anderson. However, here we will just consider their dependence on α.

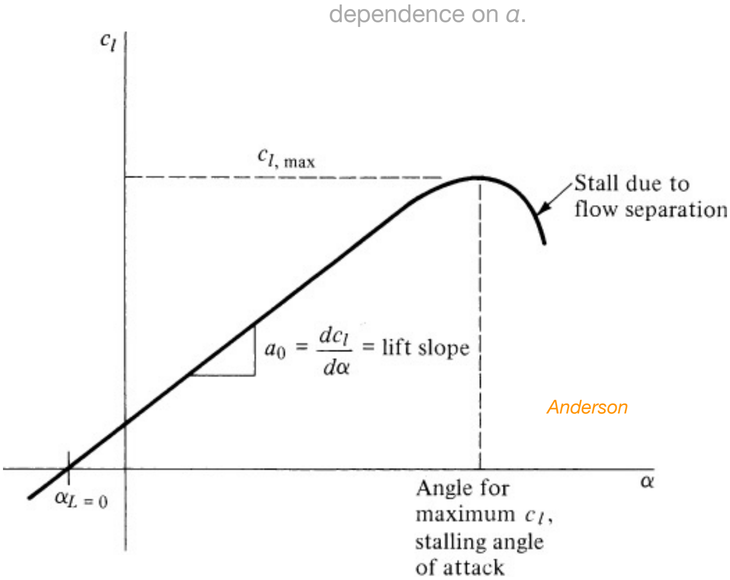
The most important consideration is how C_l depends on α.

C_l varies almost linearly on α at first, but soon starts to fall below linear, reaches a maximum and then declines.

Stall is conventionally accepted to occur angle of attack for $C_{l,\,max}$. The associated of attack is typically in the range 10° to 20°.

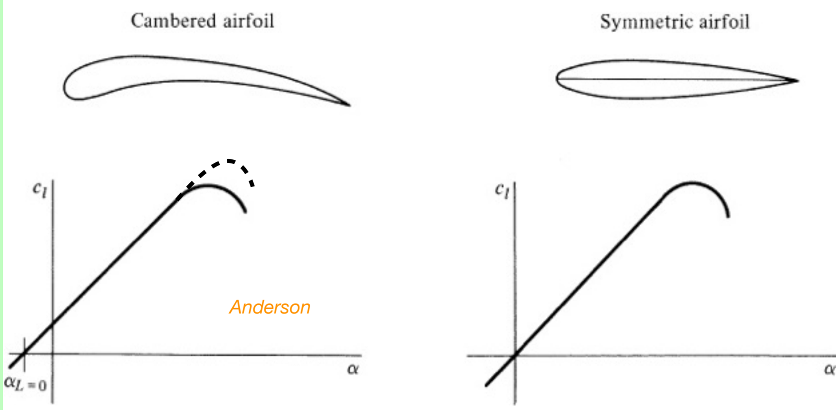
The two other leading parameters are the <u>angle of attack for zero lift</u>, $\alpha_{L=0}$ (which is negative for an airfoil with positive camber) and the <u>lift curve slope</u> $dC_l/d\alpha$.

The lift curve slope $dC_l/d\alpha$ is usually close to 2π, when α is measured in radians.



Anderson

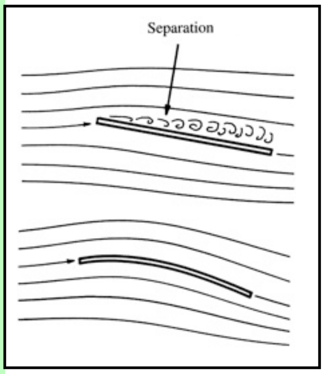
Lift, drag and moment coefficients for airfoils



Anderson

Adding mean camber to a symmetrical airfoil section moves the $C_l - \alpha$ curve to the left (so there is a positive value of lift at zero angle of attack), and also tends to increase $C_{l,max}$. <u>The overall effect is to move the curve to the left and upwards.</u>

The lift curve slope $dC_l/d\alpha$ is basically unchanged.



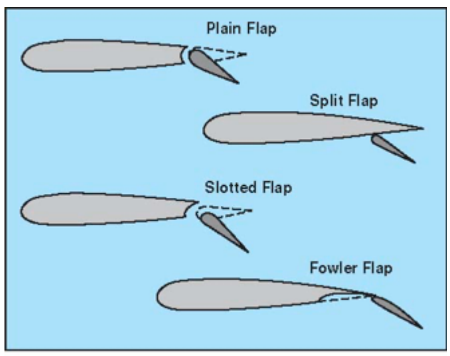
Raymer

The main reason for adding camber is to align the airfoil correctly to the oncoming flow at a positive angle of attack (and a particular positive value of C_l, the design value chosen for the aircraft).

This effect can be simply illustrated for a thin airfoil section – slight separation is avoided at one particular angle of attack.

For an airfoil of finite thickness, the effect is to minimize drag over a range of C_l centered around the aircraft design value.

To obtain even more camber and $C_{l,max}$, airfoil geometry is often varied for landing and takeoff using a flap system.



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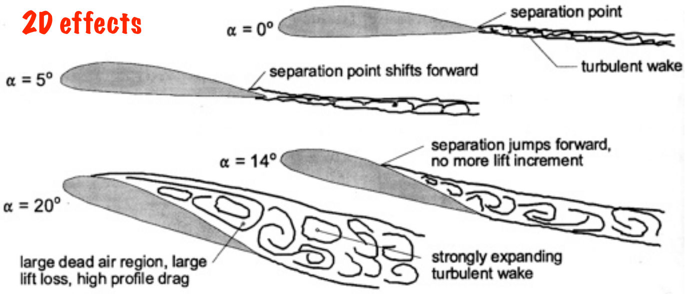
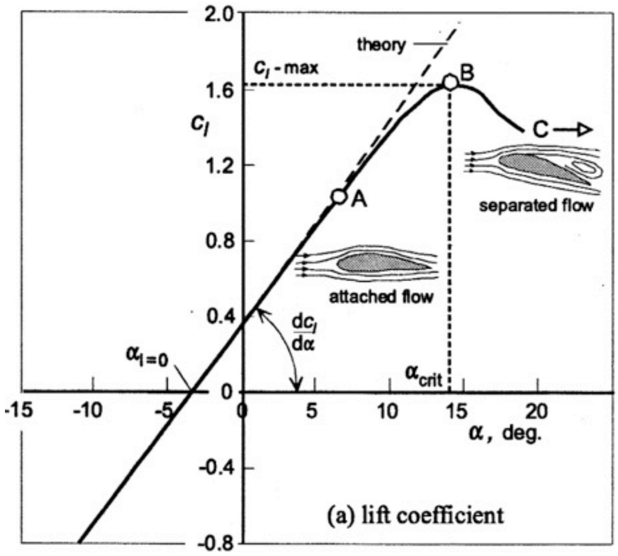
Stall phenomena

2D effects



Torenbeek & Wittenberg

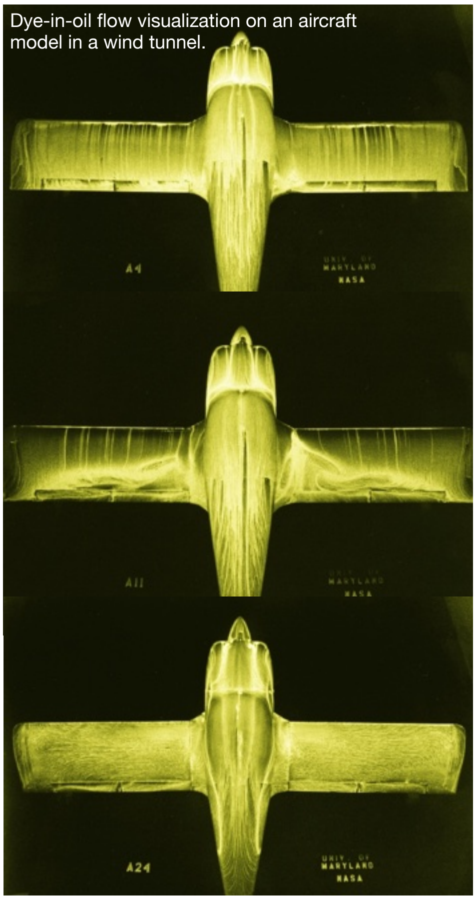


3D effects

At low angle of attack, flow stays attached over the entire top surface of the wing.

At higher angle of attack, flow starts to separate at the TE near the wing root, but remains attached near tips, i.e. partially stalled.

At still higher angle of attack, flow is completely detached over the wing upper surface. Fully stalled.


Dye-in-oil flow visualization on an aircraft model in a wind tunnel.

8

‘Never low and slow!’ – Stalls: controlled and uncontrolled

The loss of lift associated with stall can have dramatic consequences.

Lift, drag and moment coefficients for airfoils

Here is a typical presentation of lift, drag and moment coefficient data obtained using wind tunnel testing. The data show the effect of TE flap deflection, and also of Reynolds number (*R*) and surface roughness.

<u>Note that in the RH panel, data are plotted using C_l, rather than α, as the abscissa variable.</u>

Moment coefficient for the $c/4$ reference location ($C_{m,c/4}$) is reasonably constant as α is varied.

Moment coefficient for the aerodynamic center location ($C_{m,\text{ac}}$) is very similar, and is almost completely invariant.

Note that actual location of the a.c. is given and it is quite close to $c/4$ position.

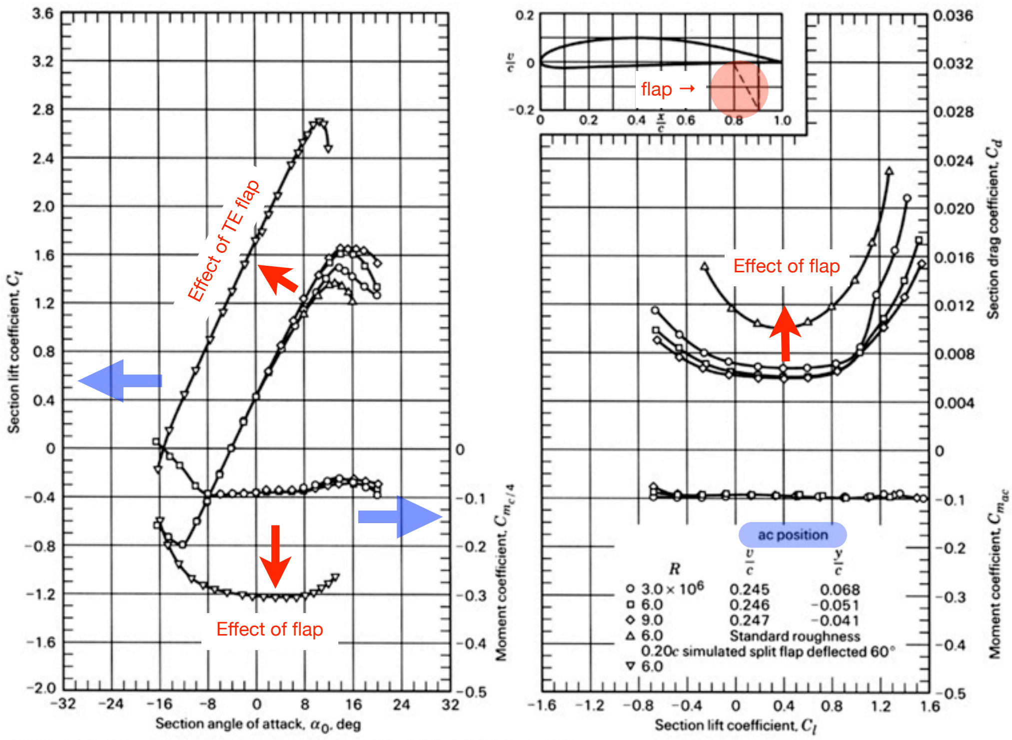


Figure 3.7 Aerodynamic characteristics of the NACA 4412 airfoil.

McCormick

Lift, drag and moment coefficients for airfoils

<u>Example: estimate lift force acting on a wing spanning 1.8m between wind tunnel walls, angle of attack 6°.</u>

Wing section NACA 4412, chord 0.4m, wind tunnel speed 100m/s, ISA sea-level conditions.

Note for later reference: the wing spans tunnel completely, so end effects are minimal.
Adequate to make an analysis based only on sectional values.

$$Re_c = \frac{\rho V c}{\mu} = \frac{1.225 \times 100 \times 0.4}{17.3 \times 10^{-6}} = 2.83 \times 10^6$$

Close enough to 3.0×10^6, lowest value shown on chart.

From chart at α = 6°, estimate C_l = 1.0, $C_{m,c/4}$ = –0.08, C_d = 0.0075. $C_{m,\text{a.c.}}$ = –0.10.

Dynamic pressure $q = \frac{1}{2}\rho V^2 = 0.5 \times 1.225 \times 100^2\,\text{Pa} = 6.125\,\text{kPa}$

Wing area $S = 1.8 \times 0.4\,\text{m}^2 = 0.72\,\text{m}^2$

<u>Lift</u> (total, not per unit width) $L = \frac{1}{2}\rho V^2 S C_l = 6.125 \times 10^3 \times 0.72 \times 1.0\,\text{N} = 4.41\,\text{kN}$

This is equivalent to the gravity force experienced by a mass of 450kg, i.e. approx. 1/2 tonne.

<u>Drag</u> $D = \frac{1}{2}\rho V^2 S C_d = 6.125 \times 10^3 \times 0.72 \times 0.0075\,\text{N} = 33.08\,\text{N}$

<u>This ‘airfoil profile’ drag includes both skin friction and pressure effects.</u>

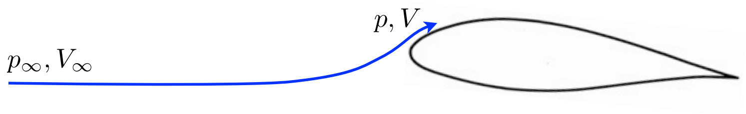
Ratio of Lift/Drag = $L/D = C_l/C_d$ = 133.3.

<u>Moment</u> $M_{c/4} = \frac{1}{2}\rho V^2 S c C_{m,c/4} = -6.125 \times 10^3 \times 0.72 \times 0.4 \times 0.08\,\text{Nm} = -141.1\,\text{Nm}$ (nose down)

This is equivalent to the moment produced by gravity force
acting on a mass of 14.4kg, with a 1m moment arm.

Pressure coefficient

In a pressure distribution, pressures will change as the free-stream speed changes. For inviscid flow we have Bernoulli's equation relating pressures around the airfoil to those in the free-stream:



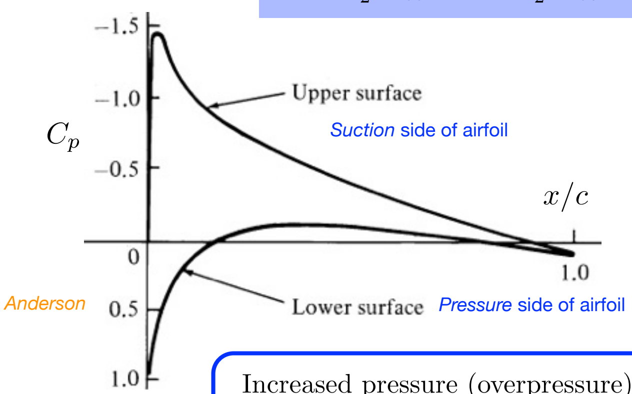
$$\tfrac{1}{2}\rho V_\infty^2 + p_\infty = \tfrac{1}{2}\rho V^2 + p$$

So $\quad p - p_\infty = \tfrac{1}{2}\rho V_\infty^2 \left[1 - (V/V_\infty)^2\right] \quad$ or $\quad p - p_\infty \propto \tfrac{1}{2}\rho V_\infty^2$

In dimensionless form:

$$C_p = \frac{p - p_\infty}{\frac{1}{2}\rho V_\infty^2} = \frac{\frac{1}{2}\rho_\infty (V_\infty^2 - V^2)}{\frac{1}{2}\rho V_\infty^2} = 1 - \left(\frac{V}{V_\infty}\right)^2$$

C_p is called a pressure coefficient.

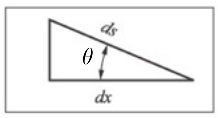


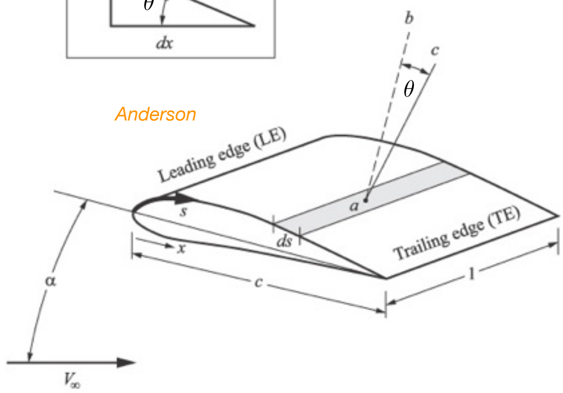
For airfoils, it is normal to plot C_p as a function of dimensionless distance along the chordline, x/c.

By convention, it is also usual to reverse the sense of C_p on the ordinate axis, so that 'suction is up'.

Increased pressure (overpressure):	$p > p_\infty$	$\iff$	$V < V_\infty$	$\rightarrow$	$C_p > 0$
Reduced pressure (suction):	$p < p_\infty$	$\iff$	$V > V_\infty$	$\rightarrow$	$C_p < 0$
Stagnation point:	$V = 0$	$\iff$	$p = p_0$	$\rightarrow$	$C_p = 1$
Undisturbed flow:	$V = V_\infty$	$\iff$	$p = p_\infty$	$\rightarrow$	$C_p = 0$

Calculating lift coefficient from pressure coefficient distribution





Force per unit width produced by pressure, acting in a direction normal to chord line on an element of upper airfoil surface is $\mathrm{d}N_u = -p_u \cos\theta \, \mathrm{d}s$

Hence total normal force per unit width from upper surface pressure is

$$N_u = -\int_{\mathrm{LE}}^{\mathrm{TE}} p_u \cos\theta \, \mathrm{d}s$$

And $\quad N = \int_{\mathrm{LE}}^{\mathrm{TE}} p_l \cos\theta \, \mathrm{d}s - \int_{\mathrm{LE}}^{\mathrm{TE}} p_u \cos\theta \, \mathrm{d}s$

Change variables (geometry) $\quad \mathrm{d}s \cos\theta = \mathrm{d}x$

$$N = \int_0^c p_l \, \mathrm{d}x - \int_0^c p_u \, \mathrm{d}x$$

Subtract freestream pressure $\quad N = \int_0^c (p_l - p_\infty) \, \mathrm{d}x - \int_0^c (p_u - p_\infty) \, \mathrm{d}x$

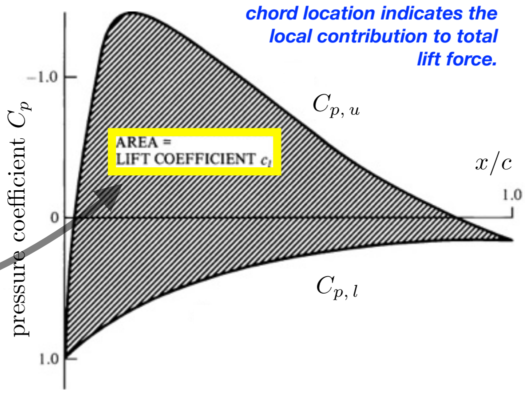
Convert to force coefficient

$$C_n = \frac{N}{\frac{1}{2}\rho V_\infty^2 c} = \frac{N}{q_\infty c} = \frac{1}{c}\int_0^c \frac{p_l - p_\infty}{q_\infty} \, \mathrm{d}x - \frac{1}{c}\int_0^c \frac{p_u - p_\infty}{q_\infty} \, \mathrm{d}x$$

As pressure coefficients $\quad C_n = \frac{1}{c}\int_0^c (C_{p,l} - C_{p,u}) \, \mathrm{d}x$

Recall $\quad L = N\cos\alpha - A\sin\alpha \quad$ i.e. $\quad C_l = C_n \cos\alpha - C_a \sin\alpha$

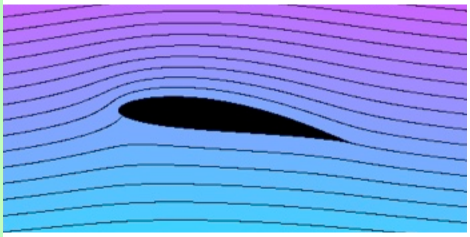
Hence for small α, $\quad C_l \approx \int_0^c (C_{p,l} - C_{p,u}) \, \frac{\mathrm{d}x}{c}$



The difference in C_p at any chord location indicates the local contribution to total lift force.

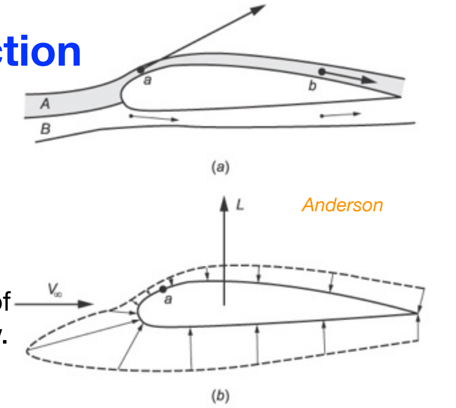
AERODYNAMICS

Mechanisms of lift production



The basic mechanism is that air flows faster over the top of an airfoil than below it, when lift is being produced.

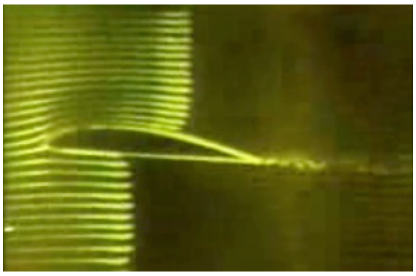
This is evident from the fact that initially equispaced streamlines get closer together over the top of a lifting airfoil, further apart below.



Note: The length of the arrows denoting pressure is proportional to $p - p_{ref}$, where p_{ref} is an arbitrary reference pressure slightly less than the minimum pressure on the airfoil.

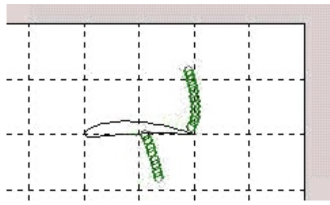
From continuity, speeds above are higher than those below. From Bernoulli's equation, this implies generally lower pressures above than below, and when the contributions are integrated around the surface of the airfoil, there is a net lift force. This is the physical mechanism of lift.

There is an equal and opposite force per unit length exerted on the fluid passing the airfoil, and momentum considerations imply that the passing air flow locally acquires a slight downwash velocity.



An argument sometimes used to explain the speed difference on the two sides is that the path around the top of the airfoil is longer and hence particles travelling over the top have to go faster in order to reach the TE at the same time as those travelling underneath but released at the same instant near the LE.

This is not supported by evidence: in fact particles travelling over the top arrive at the TE well before those travelling underneath, i.e. move faster than this argument requires.



Relative to the freestream, fluid travelling past the top of a lifting airfoil moves faster, while that travelling below moves slower.

So far, we have no means of predicting how much lift a given airfoil will produce.

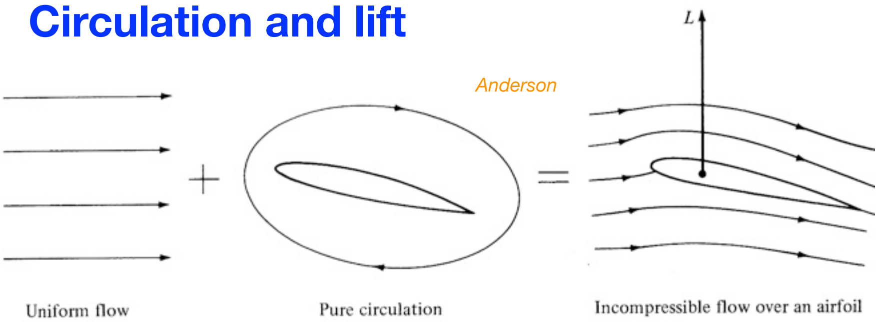
AERODYNAMICS

Circulation and lift

The relative speed-up/slow-down above/below can be interpreted as a net circulation around the airfoil.

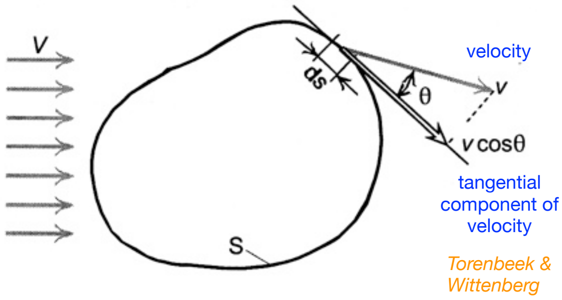
This is the physical basis of the accepted theory of lift production.

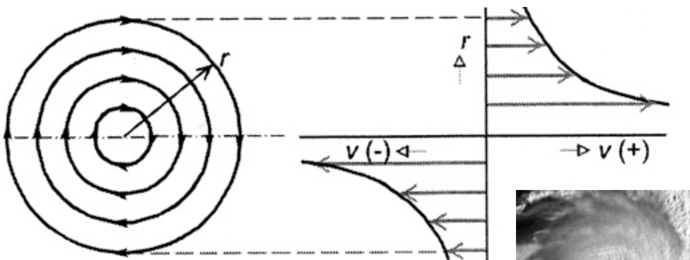
The mathematical expression of the circulation theory of lift gives extremely accurate predictions.

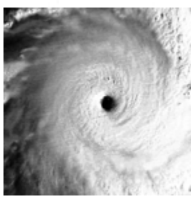


Around an arbitrary closed curve S in a 2D velocity field, the **circulation** Γ is defined as the integral around the curve of the tangential component of velocity.

$$\Gamma = \int_s V \cos\theta \, ds$$







Cyclonic flows approximate this ideal vortex.

E. g. an inviscid flow generated around a line vortex has a circulation that does not vary with radius.

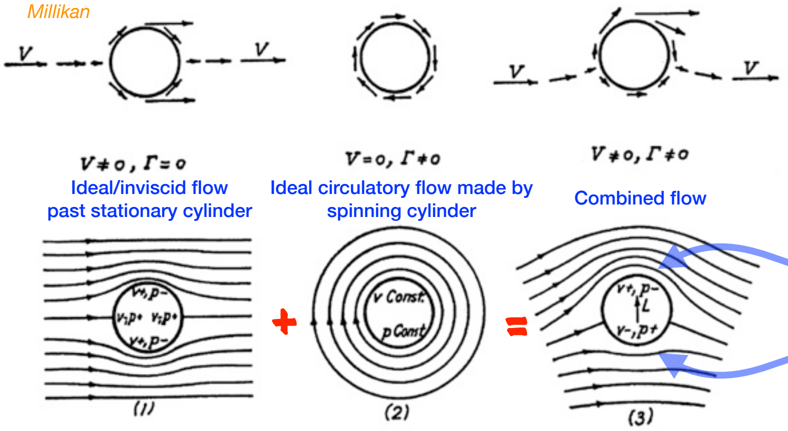
In this case $\Gamma = 2\pi r V = \text{const.}$

which means the velocity must fall inversely with radius. This is a reasonable approximation in many real vortex flows except very near to the vortex, where viscosity has an effect.

AERODYNAMICS

Lift related to circulation for flow past a spinning cylinder

Millikan

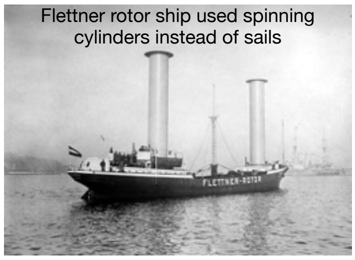


Ideal/inviscid flow past stationary cylinder

Ideal circulatory flow made by spinning cylinder

Combined flow

$\Gamma = \mathrm{const.}$



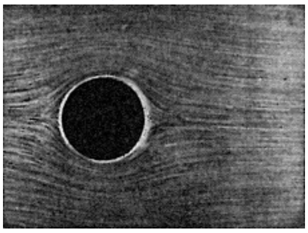
High velocity, low pressure

Low velocity, elevated pressure

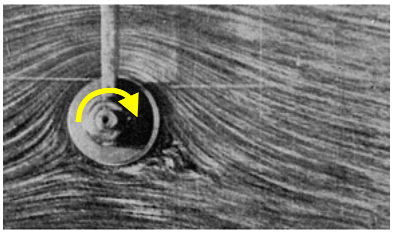
Lift force (per unit length of cylinder)

$$L = \rho V \Gamma$$

★ **Kutta-Joukowsky law**



Prandtl & Tietjens



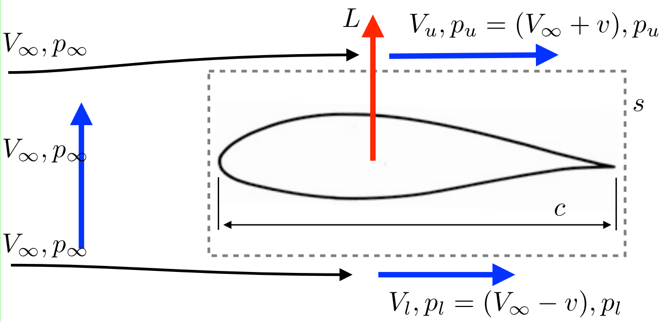
For round spinning objects (like cylinders or balls) the production of lift or side force in this way is known as the Magnus effect – often used in ball sports.

AERODYNAMICS

Lift related to circulation

A simplified (and approximate!) explanatory model assuming velocities and pressures above and below an airfoil can be characterised by single-point values.

Consider lift force <u>per unit length</u> L produced by flow past an airfoil, chord c. Local speed increments $\pm v$.



Bernoulli

$$p_u + \tfrac{1}{2}\rho V_u^2 = p_\infty + \tfrac{1}{2}\rho V_\infty^2 \qquad p_l + \tfrac{1}{2}\rho V_l^2 = p_\infty + \tfrac{1}{2}\rho V_\infty^2$$

$$p_u - p_\infty = \tfrac{1}{2}\rho(V_\infty^2 - V_u^2) \qquad p_l - p_\infty = \tfrac{1}{2}\rho(V_\infty^2 - V_l^2)$$

now

$$V_u = V_\infty + v, \quad V_u^2 = V_\infty^2 + 2V_\infty v + v^2$$
$$V_l = V_\infty - v, \quad V_l^2 = V_\infty^2 - 2V_\infty v + v^2$$

so

$$p_u - p_\infty = \tfrac{1}{2}\rho(-2V_\infty v - v^2)$$
$$p_l - p_\infty = \tfrac{1}{2}\rho(+2V_\infty v - v^2)$$

pressure difference

$$\Delta p = p_u - p_l = -4 \times \tfrac{1}{2}\rho V_\infty v = -2\rho V_\infty v$$

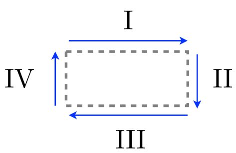
then

$$L = -\Delta p c = 2\rho V_\infty v c = \rho V_\infty 2vc$$

Circulation

$$\Gamma = \int_s V\cos\theta\,\mathrm{d}s = I_{\mathrm{I}} + I_{\mathrm{II}} + I_{\mathrm{III}} + I_{\mathrm{IV}} \quad = V_u c + 0 - V_l c + 0 \quad = (V_\infty + v)c + 0 - (V_\infty - v)c + 0 \quad = 2vc$$

integration paths



from above $L = \rho V_\infty 2vc$

$= \rho V_\infty \Gamma$

★ $$L = \rho V_\infty \Gamma$$ **Kutta-Joukowsky law**

17

Prandtl & Tietjens

Starting and stopping vortices – Prandtl's experiment

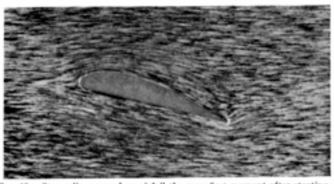


Fig. 48.—Streamlines round an airfoil the very first moment after starting.

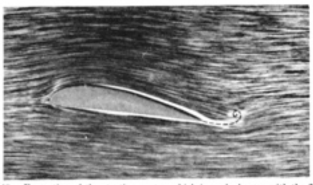


Fig. 49.—Formation of the starting vortex which is washed away with the fluid.

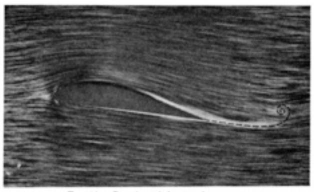


Fig. 50.—Growing of the starting vortex.

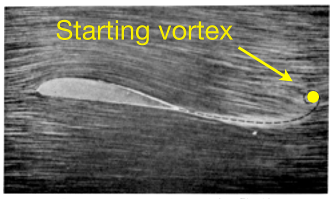


Fig. 51.—Taken somewhat later than Fig. 50.

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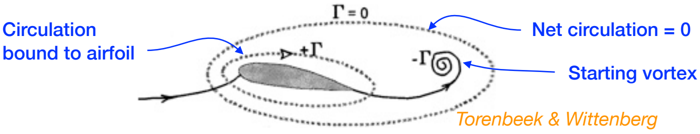
Impulsively started flow past an airfoil. Net total circulation in the fluid remains zero. The required amount of circulation travels with the airfoil. A starting vortex with equal but opposite circulation is left behind.



Torenbeek & Wittenberg

When a moving airfoil is brought suddenly to rest, its bound circulation is shed into a stopping vortex.

The pair of images below shows the result of impulsively starting, then almost immediately impulsively stopping, an airfoil. Both starting and stopping vortices are seen.

Prandtl & Tietjens

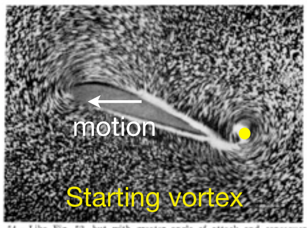


Fig. 54.—Like Fig. 52, but with greater angle of attack and consequently stronger starting vortex. Also shorter exposure of plate.

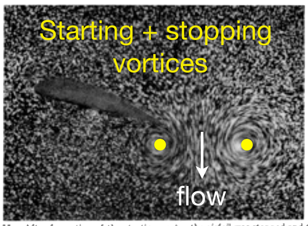
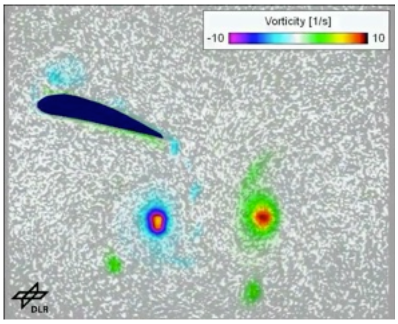


Fig. 55.—After formation of the starting vortex the airfoil was stopped and then the picture was taken.

Below we see results from the original experimental movie shown as contours of vorticity, i.e. twice the local rotation rate of fluid. This very clearly shows the generation of both starting and stopping vortices.



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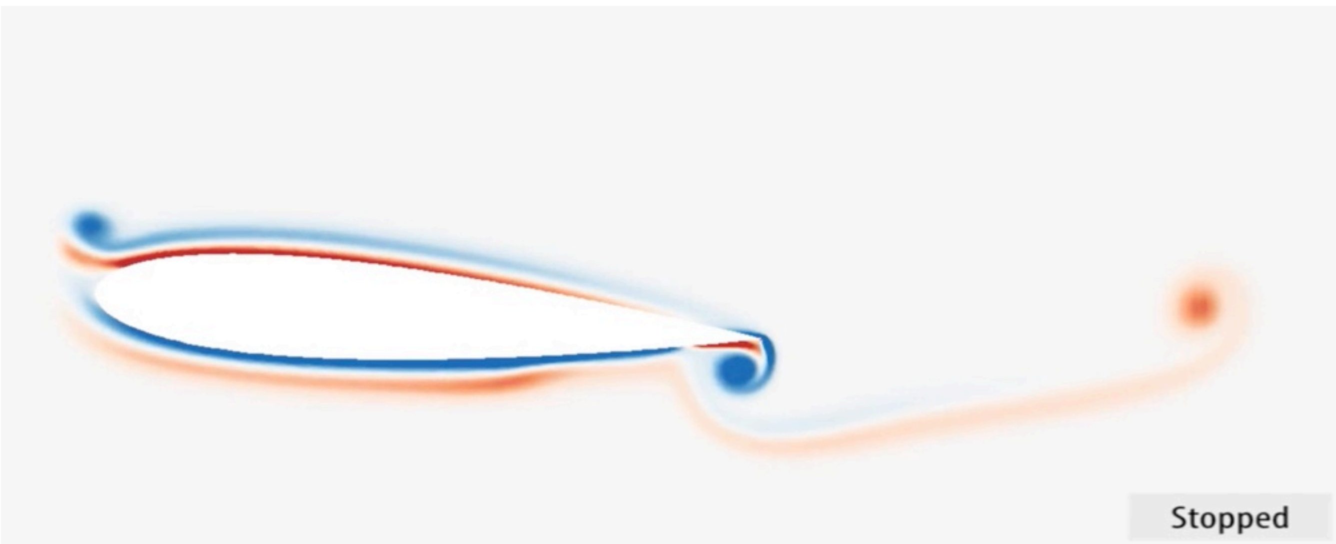
Simulation of the starting-stopping process

AERODYNAMICS



Alwin Wang FYP 2018

Kutta condition – why airfoils have sharp/cusped trailing edges

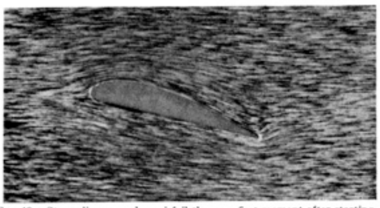
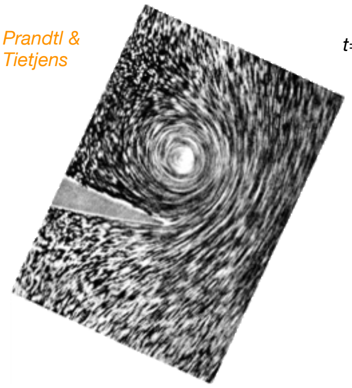


Fig. 48.—Streamlines round an airfoil the very first moment after starting.

Circulation theory on its own does not say how much circulation (and lift) will occur. **The required amount acts to provide tangential flow at the airfoil TE.** Viscous effects force a flow separation at the TE for any real flow that does not satisfy this condition. <u>Auto-correction via this separation provides viscous regulation of the airfoil's circulation.</u>

Prandtl & Tietjens



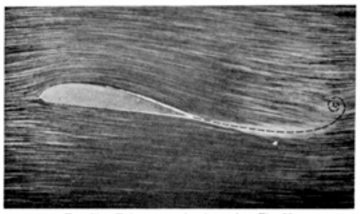
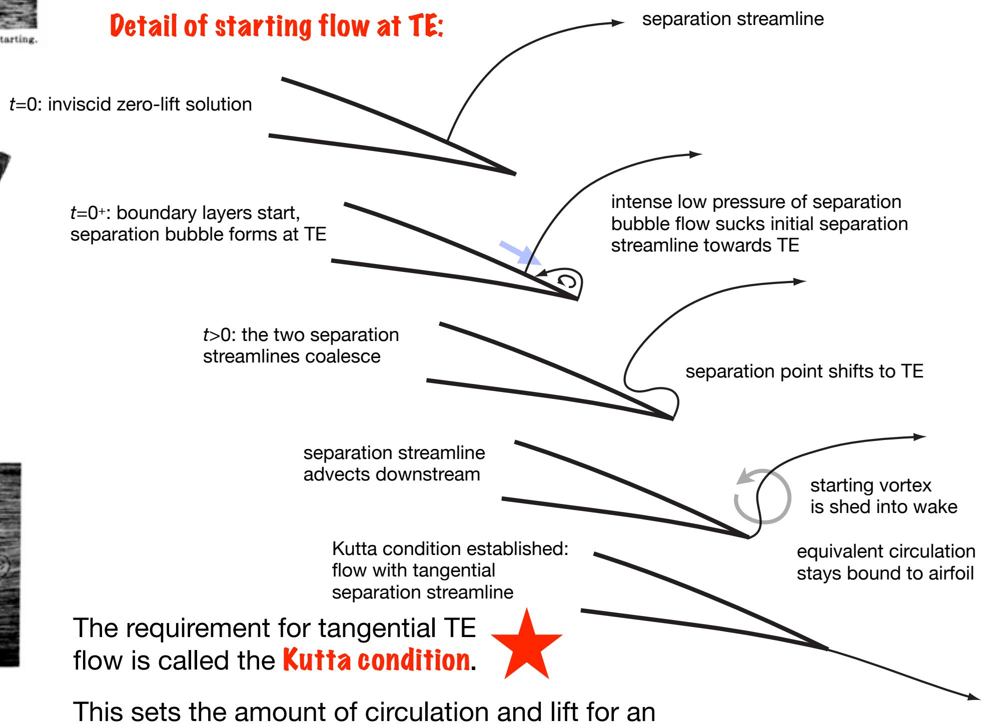


Fig. 51.—Taken somewhat later than Fig. 50.

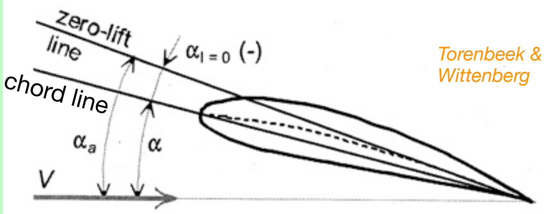
Detail of starting flow at TE:



The requirement for tangential TE flow is called the **Kutta condition**. ★

This sets the amount of circulation and lift for an airfoil of given shape and angle of attack.

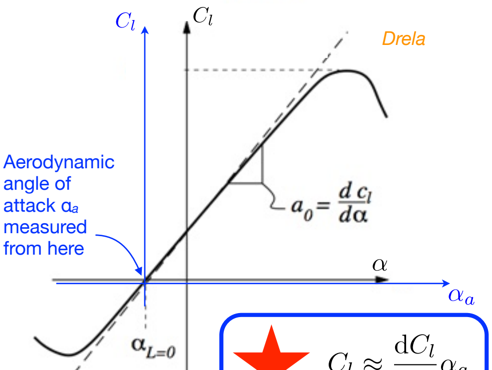
Aerodynamic angle of attack + airfoil drag polar



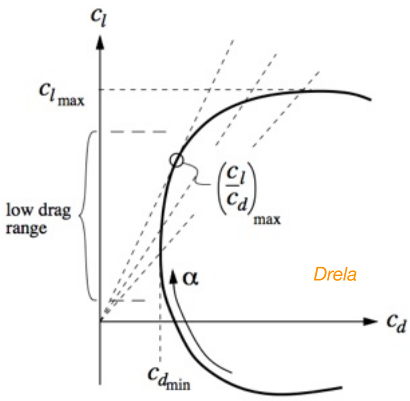
Torenbeek & Wittenberg

Airfoil performance is often characterised with respect to the **geometric angle of attack α** (measured relative to the chord line) but a more practically useful value is the **aerodynamic angle of attack α_a**, measured relative to the zero lift line.

This is equivalent to shifting the α-origin.



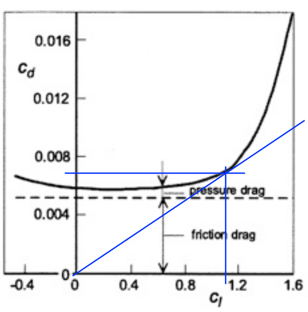
$$C_l \approx \frac{dC_l}{d\alpha}\alpha_a$$



The primary operational part of the polar is the low drag + positive lift segment of the curve.

Airfoil performance and efficiency is best characterised by the **airfoil drag polar** which shows C_l vs C_d. This shows how much useful effect (lift) we get for propulsive cost (drag). Key indicator: maximum ratio of lift to drag or $(C_l/C_d)_{max}$. On such a plot the angle of attack may be shown as a parameter but is often omitted.

The drag polar can be shown with either C_d or C_l horizontal.

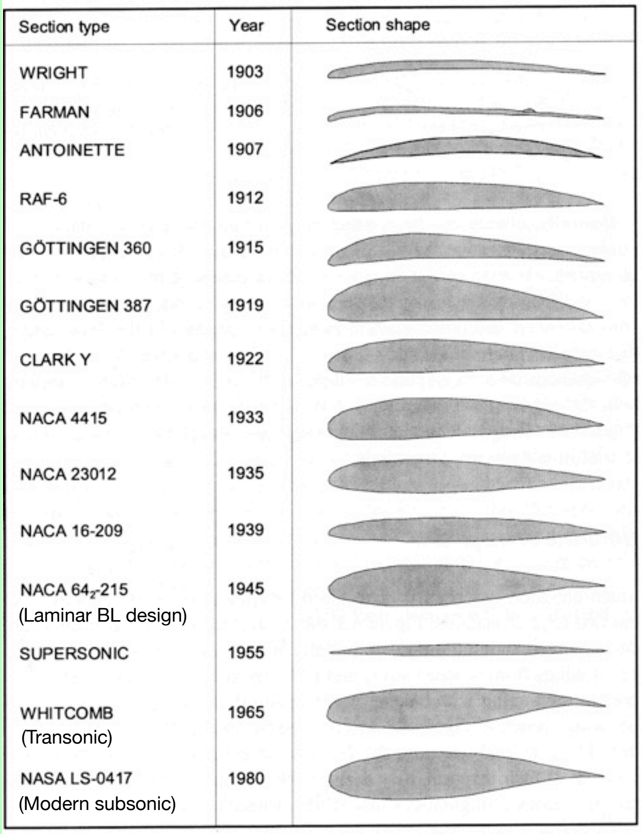


Torenbeek & Wittenberg

Over the operational range, airfoil drag is much smaller than lift, and dominated by (BL) skin friction.

Note that *airfoil* $(C_l/C_d)_{max}$ is typically quite large; here it is approximately 165.

Airfoil selection



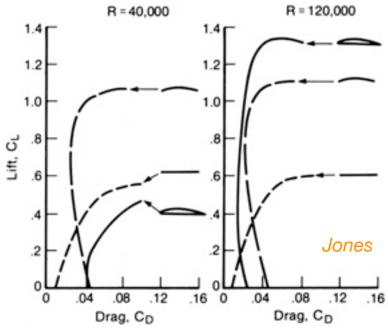
Section type	Year	Section shape
WRIGHT	1903	
FARMAN	1906	
ANTOINETTE	1907	
RAF-6	1912	
GÖTTINGEN 360	1915	
GÖTTINGEN 387	1919	
CLARK Y	1922	
NACA 4415	1933	
NACA 23012	1935	
NACA 16-209	1939	
NACA 64_2-215 (Laminar BL design)	1945	
SUPERSONIC	1955	
WHITCOMB (Transonic)	1965	
NASA LS-0417 (Modern subsonic)	1980	

Torenbeek & Wittenberg

Airfoil choice based on aerodynamic performance mainly depends on the aircraft design speed regime.

This is characterised by either Reynolds number (subsonic) or Mach number (transonic or supersonic).

At low chord Reynolds numbers, thin airfoils are best aerodynamically - reflected in early airfoil choices based on low Reynolds number wind tunnel testing. This trend is reversed at more typical aircraft *Re*:

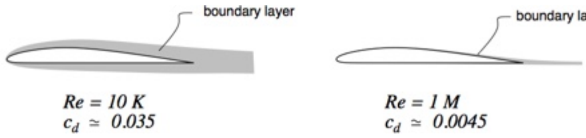


Jones

Object	*Re*
Butterfly	5 K
Pigeon	50 K
RC glider	100 K
Sailplane	1 M
Business jet	10 M
Boeing 777	50 M

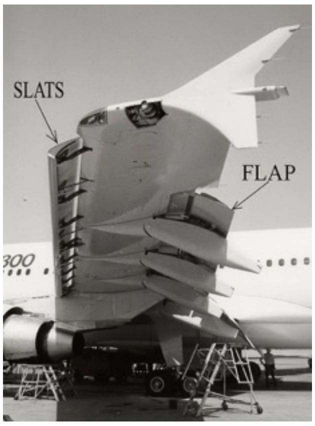
Drela

At low Mach number, airfoil design is dominated by boundary-layer related phenomena.

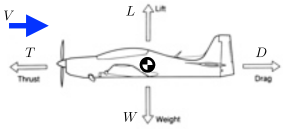


Drela

With increasing Mach number, airfoil design becomes dominated by shock-wave related phenomena.



High-lift systems



Flaps – for flying slowly

In level unaccelerated flight, lift = weight. $L = W$. $L = W = \frac{1}{2}\rho V_\infty^2 S C_L$ $\quad V_\infty = \sqrt{\frac{2}{\rho}\frac{W}{S}\frac{1}{C_L}}$

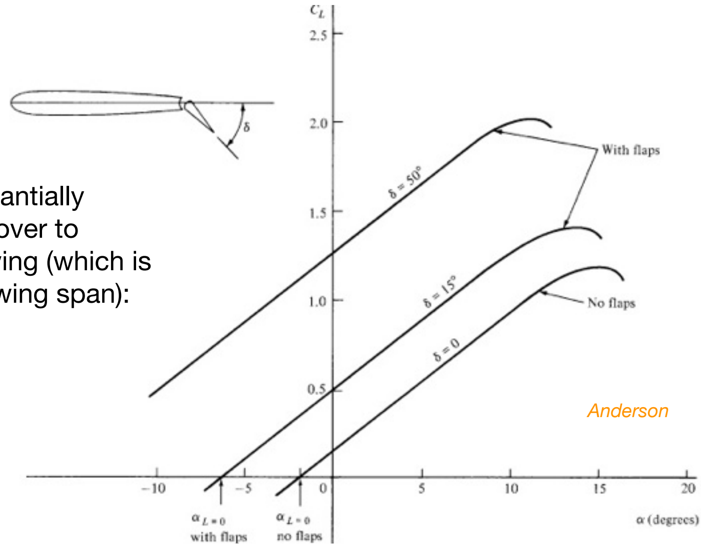
Obviously $V_{\infty,\,\min} = \sqrt{\frac{2}{\rho}\frac{W}{S}\frac{1}{C_{L,\max}}}$

Everything else equal, slowest possible flight occurs just before stall, at $C_{L,\max}$.

Slow flight is very desirable at takeoff, and especially, at landing. This reduces runway length and accident rates. If everything else is fixed, it is typically desirable to get a high value of airfoil $C_{l,\max}$. Plain-airfoil values of $C_{l,\max}$ are at best typically in the range 1.3 – 1.7, and variable airfoil geometry is employed to increase it. **Generic name for these high-lift devices: flaps**, and they can be present at both the LE and TE. The basic effect is to increase airfoil camber, but some devices also expand the effective wing area, S.

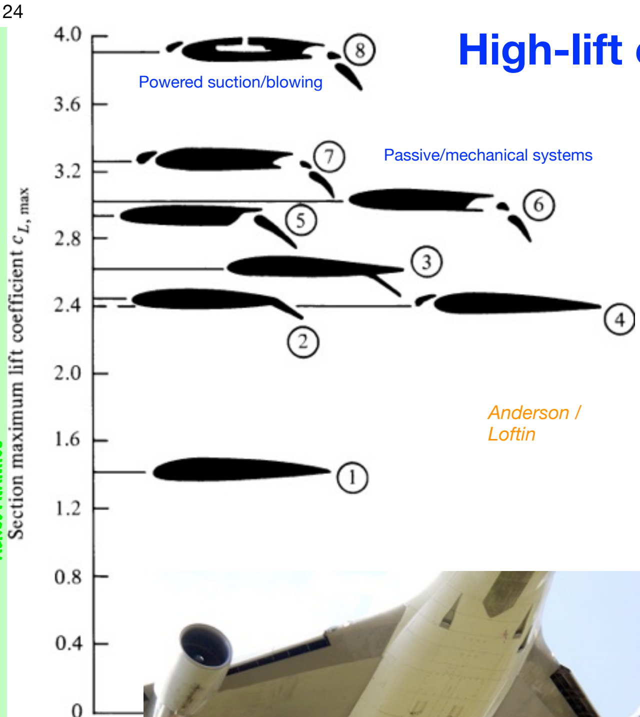
As previously noted, an increase in camber associated with a TE flap will move the Cl–α curve upwards and to the left. (A LE device will move it upward and to the right.)

One can see that $C_{l,\max}$ for the airfoil can be substantially increased. A large amount of this can be carried over to increase the overall $C_{L,\max}$ for the whole aircraft/wing (which is typically lower since devices do not cover whole wing span):

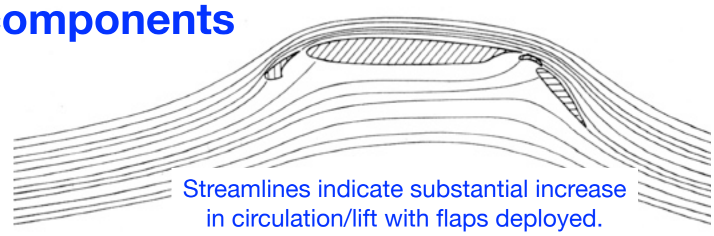


Anderson

High-lift components



Anderson / Loftin

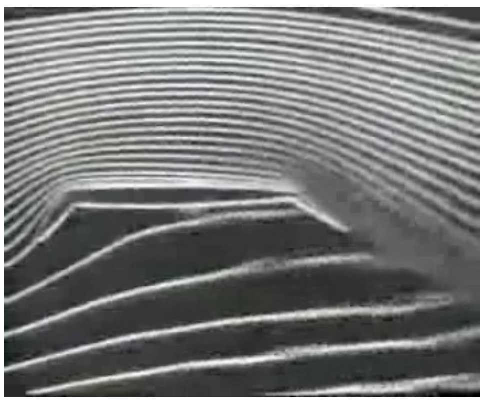


Streamlines indicate substantial increase in circulation/lift with flaps deployed.

Maximum lift coefficient values can be increased by a factor of around 2 (implying the minimum speed can be reduced by a factor 0.7071) or more, using complex (and heavy, expensive) high-lift systems.

If necessary, still greater increases may be achieved using mechanically powered blowing or suction in strategic locations ⑧.

Boeing 747 high-lift system.



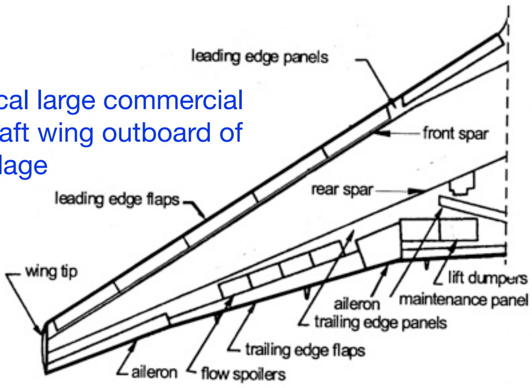
Aerodynamics of finite wings

Anderson Ch 5

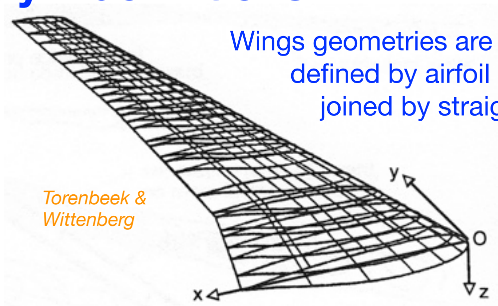
Torenbeek & Wittenberg Ch 4

Typical wing geometry + definitions

Typical large commercial aircraft wing outboard of fuselage



Wings geometries are typically defined by airfoil sections joined by straight lines.



Wings planforms are typically approximately trapezoidal (straight-tapered) in shape.

Definitions

wing area:

$$S = 2\int_0^{b/2} c(y)\,\mathrm{d}y = \frac{b}{2}c_r(1+\lambda)$$

mean geometric chord (SMC):

$$c_g = \frac{2}{b}\int_0^{b/2} c(y)\,\mathrm{d}y = \frac{S}{b} = \frac{c_r}{2}(1+\lambda)$$

mean aerodynamic chord (MAC):

$$\bar{c} = \frac{2}{S}\int_0^{b/2} c^2(y)\,\mathrm{d}y = \frac{2c_r}{3}\frac{1+\lambda+\lambda^2}{1+\lambda}$$

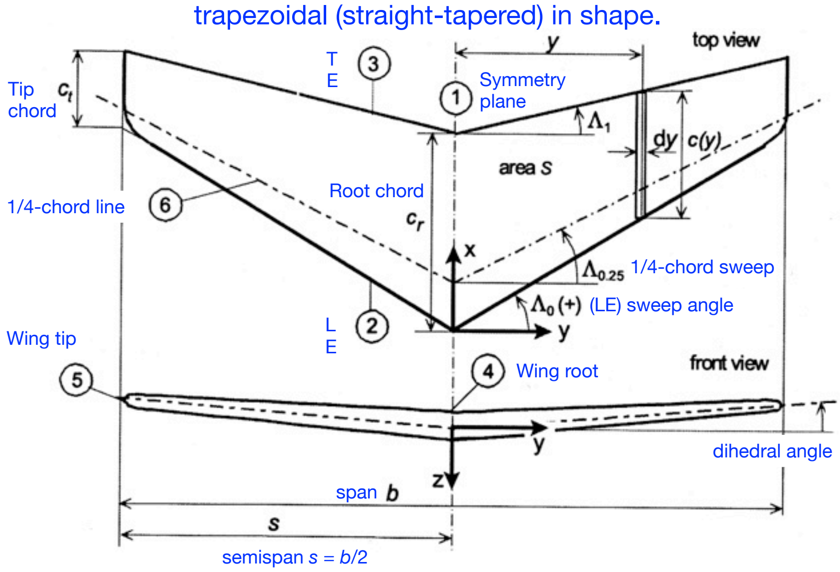
aspect ratio:

$$A = \frac{b}{c_g} = \frac{b^2}{S}$$

taper ratio:

$$\lambda = c_t/c_r$$

for trapezoidal planforms



Lift, drag & moment on finite wings

All real wings are of finite span *b*, with area *S*. Definitions of coefficients change a little compared to airfoil definitions, recognizing that we now have forces and moments, rather than forces and moments *per unit length*. Where an area is needed in the non-dimensionalization, we use wing reference area *S*. We use uppercase subscripts to denote whole-wing or whole-aircraft values.

Lift coefficient $$C_L = \frac{L}{\frac{1}{2}\rho V_\infty^2 S}$$ $[C_L]$ = [force / pressure×area] = [1]

Drag coefficient $$C_D = \frac{D}{\frac{1}{2}\rho V_\infty^2 S}$$

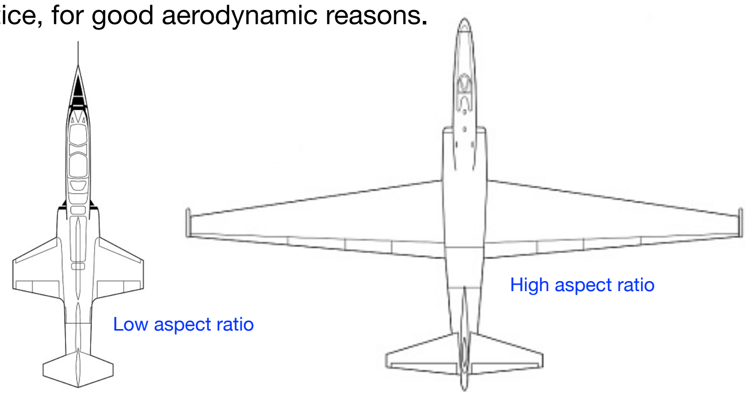
Moment coefficient $$C_M = \frac{M}{\frac{1}{2}\rho V_\infty^2 S\bar{c}}$$ $\bar{c}$ is mean aerodynamic chord.

<u>The central questions are: are these coefficients greater, or less than, those for the airfoil itself, and if different, by how much?</u>

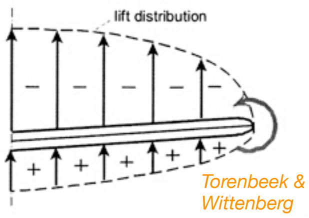
For simplicity we will ignore the fact that the airfoil could change along the span, and that the wing could be twisted from root to tip such that the geometric angle of attack varies along the span, too. Both these things often occur in practice, for good aerodynamic reasons.

Other than that, the central feature of a finite wing is how long and skinny it is, or what its span is compared to its average chord. This ratio is called the <u>wing aspect ratio</u>, given the symbol *A*.

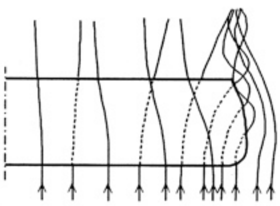
$$A = \frac{b}{c_g} = \frac{b^2}{S}$$



Finite lifting wings trail vortices behind them



(a) Flow around the tips (front view)



(b) Cross flow (top view)

Wing-tip vortices are driven by pressure differential between the upper and lower sides of a wing, associated with production of lift.

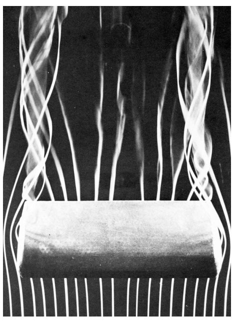
1: Pressure difference AND lift per unit span must fall to zero at wing tips.

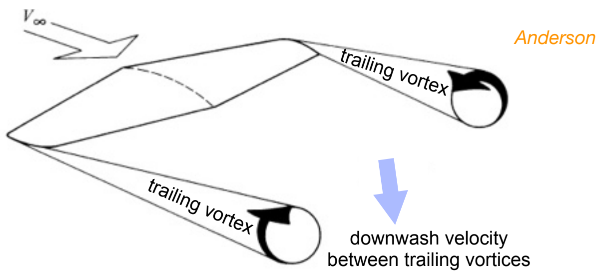
2: If wing is not producing lift there are no tip vortices.

3. A <u>downwash velocity</u> is produced by tip vortices.

van Dyke

Side view





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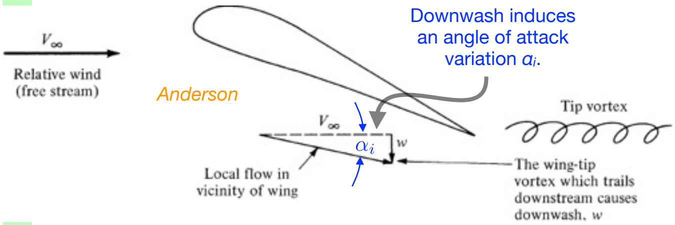
Tip-vortex downwash velocity creates drag



The downwash velocity induced by the tip vortices changes the flow near the wing, so that the effective angle of attack of each wing section is reduced by an amount called the **induced angle of attack, α_i.**

This reduces the overall lift below what would be expected from the sectional value at the same geometric angle of attack.

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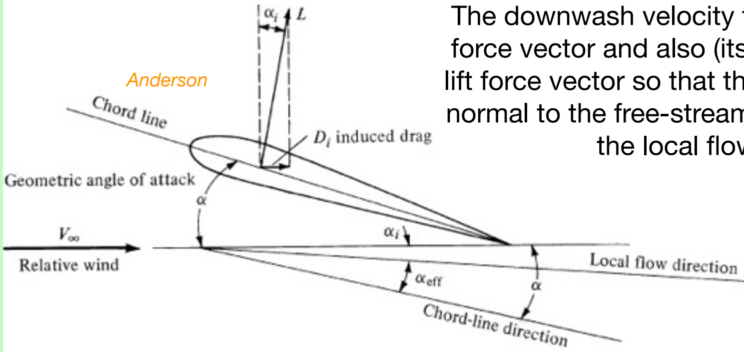
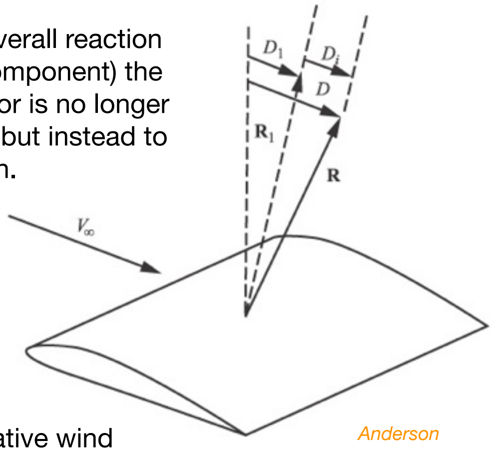
More importantly, downwash introduces an extra source of drag that is not present for an infinite wing.



The downwash velocity tilts the overall reaction force vector and also (its major component) the lift force vector so that the lift vector is no longer normal to the free-stream velocity but instead to the local flow direction.



This means that there is now a component of lift force acting in the relative wind direction – i.e. a proportion of the lift now acts as a drag, called the (lift) **induced drag**.

$D_i = L \sin\alpha_i$ or for small α_i, $D_i = L\alpha_i$

We still don't know what value α_i has. We can turn to either observation or theory to find out.

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Take 1: Prandtl's initial (imperfect) mathematical model

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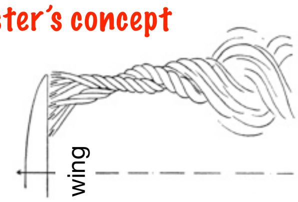
Lanchester's concept



Recall relationship between circulation and swirl velocity: $\Gamma = 2\pi r V = \text{const.}$

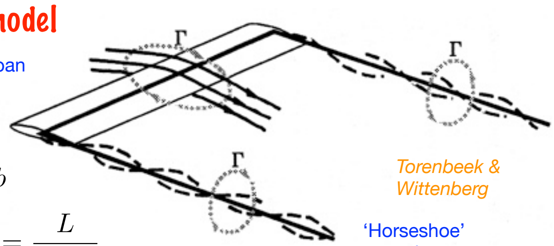
Prandtl's initial model

$\Gamma = \text{const.}$ along span

$L \approx L'b = \rho V_\infty \Gamma b$

Total lift force ≈ Lift/unit length × span

or $\Gamma = \dfrac{L}{\rho V_\infty b}$



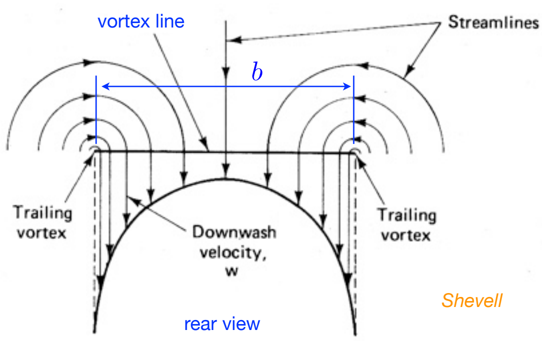
Far downstream of wing, downwash velocity mid-span is $w = 2\dfrac{\Gamma}{2\pi b/2} = \dfrac{2\Gamma}{\pi b}$ = 2× value induced by single vortex

The downwash angle far downstream, $\epsilon \approx \dfrac{w}{V_\infty} = \dfrac{2L}{\pi\rho V_\infty^2 b^2}$

Now $L = \frac{1}{2}\rho V_\infty^2 S C_L$ or $C_L = \dfrac{L}{\frac{1}{2}\rho V_\infty^2 S}$ and $A = \dfrac{b^2}{S}$ Hence $\epsilon \approx \dfrac{C_L}{\pi A}$

This gives correct downwash angle scaling but details of this single horseshoe vortex model are physically unrealistic.

E.g. it predicts infinite downwash velocities near wing tips.

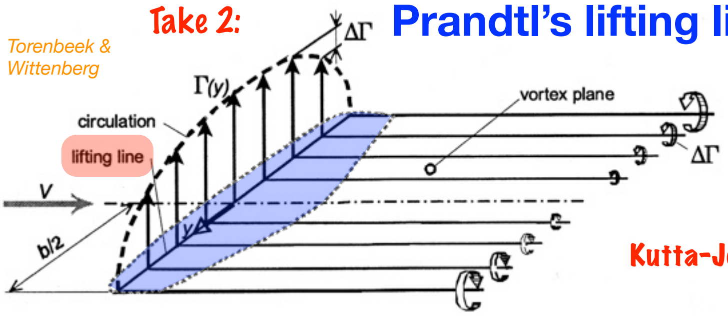


Also, the circulation (or lift force per unit length) cannot have a finite value at the wing tips, since the pressure differential has to drop to zero there.

Take 2:

Prandtl's lifting line theory

Torenbeek & Wittenberg



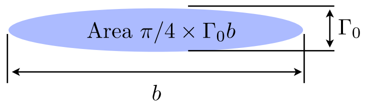
A better alternative is to let circulation (i.e. lift) vary along the span, and use a continuous array of horseshoe vortices.

$$\Gamma = \Gamma(y)$$

Kutta-Joukowsky law $$L = \rho V_\infty \int_{-b/2}^{b/2} \Gamma(y)\,\mathrm{d}y$$

It can be shown that the most efficient circulation distribution is elliptical: $\Gamma(y) = \Gamma_0 \sqrt{1 - \left(\frac{y}{b/2}\right)^2}$

In which case $L = \rho V_\infty \int_{-b/2}^{b/2} \Gamma_0 \sqrt{1 - \left(\frac{y}{b/2}\right)^2}\,\mathrm{d}y = \frac{\pi}{4}\rho V_\infty b \Gamma_0$



And for which $C_L = \frac{L}{\frac{1}{2}\rho V_\infty^2 S} = \frac{\pi}{2}\frac{\Gamma_0 b}{V_\infty S} = \frac{\pi}{2}\frac{\Gamma_0}{V_\infty c_g}$

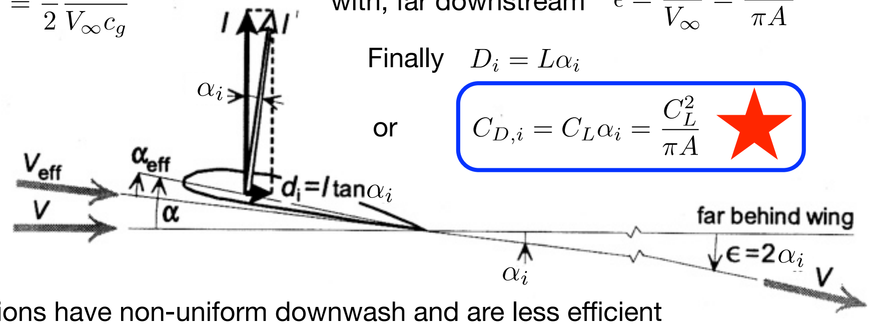
with, far downstream $\epsilon = \frac{w}{V_\infty} = \frac{2C_L}{\pi A}$

Finally $D_i = L\alpha_i$

or $$C_{D,i} = C_L \alpha_i = \frac{C_L^2}{\pi A}$$ ★

At the wing (i.e. lifting line) $\alpha_i = \epsilon/2 = \frac{C_L}{\pi A}$

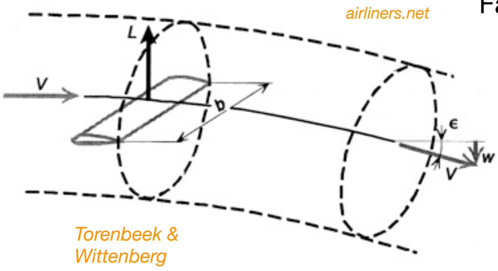
<u>For this elliptical case,</u>
<u>α_i (and w) is constant along the span.</u>



All other spanwise circulation distributions have non-uniform downwash and are less efficient i.e. produce more induced drag for a given aspect ratio.

Approximate momentum-balance explanation of induced drag

airliners.net



Torenbeek & Wittenberg

Assume freestream pressure all around the control volume.

Aircraft lift is produced by imparting a downwash velocity *w* to a mass flow rate of air.

$$L = \dot{m}w$$

For a finite wing the cross-section of air 'influenced' by the aircraft is large and scales with wingspan squared (not the wing area), b^2.
<u>This is a key assumption supported by visual evidence.</u>

$$\dot{m} = \rho V_\infty \frac{\pi}{4} b^2$$ Assuming a circle of diameter *b*.

Far downstream we have $w = V_\infty \sin\epsilon \approx V_\infty \epsilon$

$$L = \tfrac{1}{2}\rho V_\infty^2 C_L S = \dot{m}w = \rho V_\infty \frac{\pi}{4} b^2 w = \rho V_\infty^2 \frac{\pi}{4} b^2 \epsilon$$

$$\epsilon = \frac{2C_L S}{\pi b^2} = \frac{2C_L}{\pi A}$$

For horizontal momentum balance, the downstream horizontal velocity component is $V_\infty \cos\epsilon$ and the reduction is

$$V_\infty(1 - \cos\epsilon) \approx \tfrac{1}{2}V_\infty \epsilon^2 = \tfrac{1}{2}w\epsilon$$

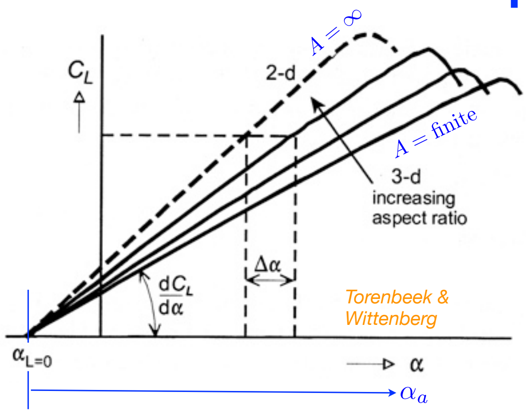
So induced drag $D_i = \dot{m}\tfrac{1}{2}w\epsilon = \tfrac{1}{2}L\epsilon$ or in dimensionless form $C_{D,i} = C_L \frac{\epsilon}{2} = \frac{C_L^2}{\pi A}$ ★ as before.

Note that we can <u>reduce induced drag *coefficient*</u> either by increasing wing aspect ratio or reducing C_L.

Alternatively, re-dimensionalising and using $L = W$ in level flight: $\frac{D_i}{W} = \frac{1}{\frac{1}{2}\rho V_\infty^2 \pi}\frac{W}{b^2}$ *W/b* is called the <u>span loading</u>.

I.e. we can <u>reduce induced drag</u> by flying faster, lower, by reducing weight, or adding wingspan.

Finite wing lift



Downwash doesn't just add drag. It also reduces lift at a given geometric (or aerodynamic) angle of attack, since the effective angle of attack is lowered.

$C_L = C_l \approx \frac{\mathrm{d}C_l}{\mathrm{d}\alpha}\alpha_a$ for infinite/2D wing, or an airfoil.

The effective angle of attack $\alpha_{\text{eff}} = \alpha_a - \alpha_i$

Suppose the wing is has an elliptical chord distribution *c*(*y*), has the same airfoil section along the span, and has no twist (change in geometric angle along the span).

In this case the circulation distribution *Γ*(*y*) is also elliptical at all angles of attack, and the downwash angle is constant along the span: wing lift coefficient is the same as the airfoil's.

$$C_L = C_l = \frac{\mathrm{d}C_l}{\mathrm{d}\alpha}\alpha_{\text{eff}} = \frac{\mathrm{d}C_l}{\mathrm{d}\alpha}(\alpha_a - \alpha_i) = \frac{\mathrm{d}C_l}{\mathrm{d}\alpha}\left(\alpha_a - \frac{C_L}{\pi A}\right)$$

Rearrange $$C_L = \alpha_a \frac{\mathrm{d}C_l/\mathrm{d}\alpha}{1 + (\mathrm{d}C_l/\mathrm{d}\alpha)/(\pi A)} \equiv \alpha_a \frac{\mathrm{d}C_L}{\mathrm{d}\alpha_a}$$

This is the 2D (airfoil) lift curve slope. (Which is a constant.)

Hence $$\frac{\mathrm{d}C_L}{\mathrm{d}\alpha} = \frac{\mathrm{d}C_l/\mathrm{d}\alpha}{1 + (\mathrm{d}C_l/\mathrm{d}\alpha)/(\pi A)}$$

For thin airfoils, theory shows $\frac{\mathrm{d}C_l}{\mathrm{d}\alpha} = 2\pi$, reasonable for most airfoils. Hence $$\frac{\mathrm{d}C_L}{\mathrm{d}\alpha} \approx \frac{2\pi}{1 + 2/A}$$

This simple model works well for moderate-to-high aspect ratio straight wings.

This shows that the lift-curve slope falls with reducing aspect ratio. C_{Lmax} is reduced too.

Proof of Prandtl's concepts

Prandtl and his students performed a set of wind tunnel experiments with wings with same airfoils but different aspect ratios, varied α, measured C_L and C_D. This was a test of the lifting-line theory.

1. $C_D \approx C_d + \frac{C_L^2}{\pi A}$ so at fixed C_L but different *A*: $C_{D,1} \approx C_d + \frac{C_L^2}{\pi A_1}$ and $C_{D,2} \approx C_d + \frac{C_L^2}{\pi A_2}$

   Airfoil profile drag + Induced drag

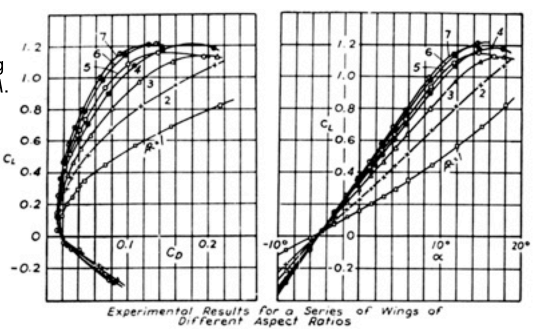
$$C_{D,1} \approx C_{D,2} + \frac{C_L^2}{\pi}\left(\frac{1}{A_1} - \frac{1}{A_2}\right)$$

2. $C_L \approx \frac{\mathrm{d}C_l}{\mathrm{d}\alpha}(\alpha_a - \alpha_i) = \frac{\mathrm{d}C_l}{\mathrm{d}\alpha}\left(\alpha_a - \frac{C_L}{\pi A}\right) = \frac{\mathrm{d}C_l}{\mathrm{d}\alpha}\left(\alpha - \alpha_{L=0} - \frac{C_L}{\pi A}\right)$

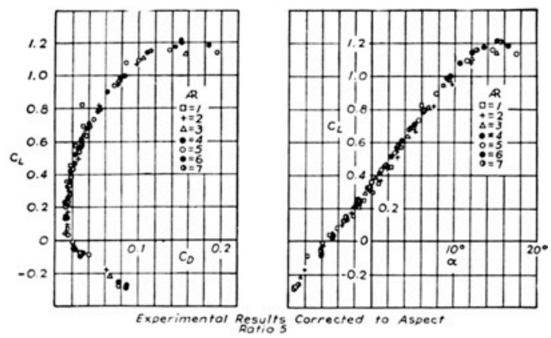
   so $C_L = \frac{\mathrm{d}C_l}{\mathrm{d}\alpha}\left(\alpha_1 - \alpha_{L=0} - \frac{C_L}{\pi A_1}\right) = \frac{\mathrm{d}C_l}{\mathrm{d}\alpha}\left(\alpha_2 - \alpha_{L=0} - \frac{C_L}{\pi A_2}\right)$

$$\alpha_1 = \alpha_2 + \frac{C_L}{\pi}\left(\frac{1}{A_1} - \frac{1}{A_2}\right)$$

Experiments at various wing aspect ratios *A*.



Corrected to a common aspect ratio A=5.



Prandtl

A classic example of using a well-designed experiment to test a theory

The wing drag polar

We have already encountered the airfoil drag polar, the relationship between C_d and C_l.

The overall drag of an isolated wing contains contributions both from its airfoil profile (boundary layer skin friction and pressure drag - combined into the 'profile drag' coefficient) and induced drag.

If we assume that the airfoil profile is the same along the wing then a reasonable approximation for the whole-wing drag coefficient is

$$C_D \approx C_d + C_{D,i}$$

Total wing drag — Airfoil profile drag — Lift-induced drag

where we have seen that $C_{D,i} \approx \dfrac{C_L^2}{\pi A}$, exact for an elliptical lift distribution.

In practice the induced drag for a wing is larger than this owing to non-ideal lift distributions, and we put

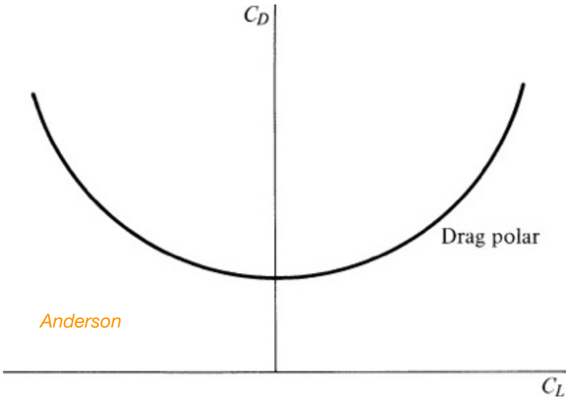
$$C_D \approx C_d + \frac{C_L^2}{\pi A\, u}$$

Total drag — Profile drag — Induced drag

where $0 < u < 1$ is the 'span efficiency' factor.

This is called the wing drag polar and is a first approximation to the whole-aircraft drag polar.

We note that the profile drag coefficient C_d could be a function of lift coefficient. However, to good degree of approximation, the overall shape is quite close to parabolic, provided C_L is not so large that stall is being approached.



Wing drag has a surprise

$$C_D \approx C_d + \frac{C_L^2}{\pi A\, u}$$

In level steady flight lift = weight. $L = \frac{1}{2}\rho V^2 S C_L = W$ $\qquad C_L = \dfrac{2}{\rho}\dfrac{W}{S}\dfrac{1}{V^2}$

We note that the relative importance of the induced drag contribution falls as C_L is reduced. In level flight, this generally means the contribution of induced drag falls with increased aircraft speed or lower altitude.

$$C_D = C_d + \frac{C_L^2}{\pi A u} \qquad D = \tfrac{1}{2}\rho V^2 S C_D = \tfrac{1}{2}\rho V^2 \left(C_d + \frac{C_L^2}{\pi A u}\right) \qquad C_L^2 = \left(\frac{2}{\rho}\frac{W}{S}\right)^2 \frac{1}{V^4}$$

$$D = \tfrac{1}{2}\rho V^2 S \left[C_d + \frac{1}{\pi A u}\left(\frac{2}{\rho}\frac{W}{S}\right)^2 \frac{1}{V^4}\right]$$

$$= \tfrac{1}{2}\rho S \left[V^2 C_d + \frac{1}{V^2}\frac{1}{\pi A u}\left(\frac{2}{\rho}\frac{W}{S}\right)^2\right]$$

term $\propto V^2$ $\qquad$ term $\propto \dfrac{1}{V^2}$

Drag that FALLS with speed is rather unusual in mechanics.

Wing drag polar example calculation

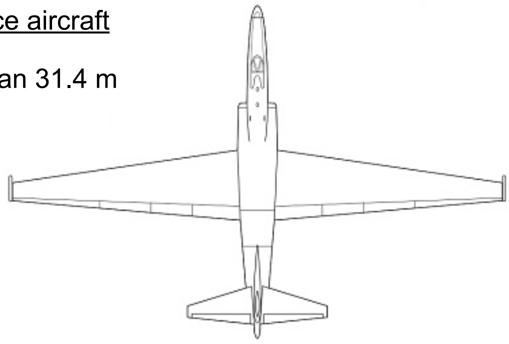
$C_D \approx C_d + \frac{C_L^2}{\pi A u}$ In level steady flight lift = weight. $L = \frac{1}{2}\rho V^2 S C_L = W$ $C_L = \frac{2}{\rho}\frac{W}{S}\frac{1}{V^2}$

We note that the relative importance of the induced drag contribution falls as C_L is reduced. In level flight, this generally means the contribution of induced drag falls with increased aircraft speed or lower altitude.

Drag that FALLS with speed is rather unusual in mechanics.

Example: ER2 reconnaissance aircraft

Wing area 92.9 m^2, wing span 31.4 m
Max TO weight 177.6 kN
Cruise speed 180 m/s
Working altitude 20 km
Wing section NACA 63_a409



Estimate drag at SL and working altitude, assuming same cruise speed and aircraft weight at both altitudes.

Wing area S = 92.9 m^2, wing span b = 31.4 m Aspect ratio A = b^2/S = 10.6. Reasonable value of u = 0.9.

Air density: at SL: ρ = 1.225 kg/m^3; at 20km: ρ = 0.08891 kg/m^3.

Wing loading $\frac{W}{S} = \frac{177.6 \times 10^3}{92.9}\,\mathrm{Pa} = 1.911\,\mathrm{kPa}$

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Wing drag polar example calculation

Wing area S = 92.9m^2, wing span b = 31.4m Aspect ratio A = b^2/S = 10.6. Reasonable value of u = 0.9.

Air density: at SL: ρ = 1.225kg/m^3; at 20km: ρ = 0.08891kg/m^3.

Level flight: $L = W = \frac{1}{2}\rho V^2 S C_L$ or $C_L = \frac{2}{\rho}\frac{W}{S}\frac{1}{V^2}$ and $\frac{W}{S} = \frac{177.6 \times 10^3}{92.9}\,\mathrm{Pa} = 1.911\,\mathrm{kPa}$

At SL, cruise speed, $C_L = \frac{2}{1.225} \times \frac{1911}{180^2} = 0.0963$ $C_{D,i} = \frac{C_L^2}{\pi A u} = \frac{0.0963^2}{\pi \times 10.6 \times 0.9} = 0.00031$

At 20km, cruise, $C_L = \frac{2}{0.08891} \times \frac{1911}{180^2} = 1.33$ $C_{D,i} = \frac{1.33^2}{\pi \times 10.6 \times 0.9} = 0.0590$ **190 times larger**

Recall $D = \frac{1}{2}\rho V^2 S C_D$ and $C_D \approx C_d + C_{D,i} \approx C_d + \frac{C_L^2}{\pi A u}$

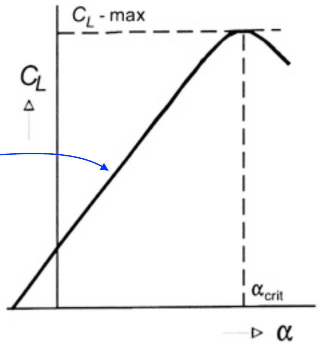
Reasonable minimum value of C_d for this airfoil section: C_d = 0.004, assume OK for both conditions.

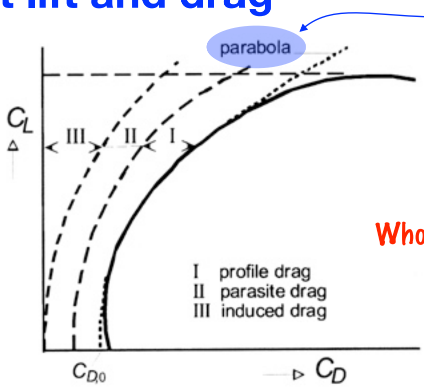
	C_d	$C_{D,i}$	C_D	profile drag	induced drag	total drag
SL cruise	0.004	0.00031	0.00431	7374N	572N	7946N
20km cruise	0.004	0.059	0.063	535N	7895N	8430N

Note that while the total drag force stays nearly the same (coincidence) the relative contributions of the profile and induced drag are almost completely the opposite in the two cases.

AERODYNAMICS

Whole-aircraft lift and drag





Whole-aircraft Drag Polar

Whole-aircraft lift L and lift coefficient $C_L = \dfrac{L}{q_\infty S} = \dfrac{L}{\frac{1}{2}\rho V_\infty^2 S}$ is based on the wing reference area S.

Whole aircraft drag D contains a number of contributions, whose relative importance varies with C_L.

For a subsonic aircraft these contributions are

- I: D_p, Profile drag of lift-producing components (wing and horizontal stabilizer)
- II: D_{par}, Parasitic drag of non-lifting components (fuselage, engine nacelles, vertical tail ...).
- III: D_i, Induced drag of lifting components (wing and horizontal stabilizer).

The aerodynamic distinction between D_p and D_i, both for lift-producing components, is that D_p is produced by boundary-layer effects and is dependent on viscosity (or Reynolds number), whereas D_i is caused by flows that are effectively inviscid (and are comparatively easy to compute).

$$D = D_p + D_{\rm par} + D_i \qquad \text{with} \qquad C_D = \frac{D}{q_\infty S} = \frac{D}{\frac{1}{2}\rho V_\infty^2 S} = C_{D_p} + C_{D_{\rm par}} + C_{D_i}$$

The parasitic drag contribution was not included in our previous discussion of wing drag.

Whole-aircraft drag polar

The whole-aircraft drag polar is the single most important concept used in aircraft performance analysis and design.

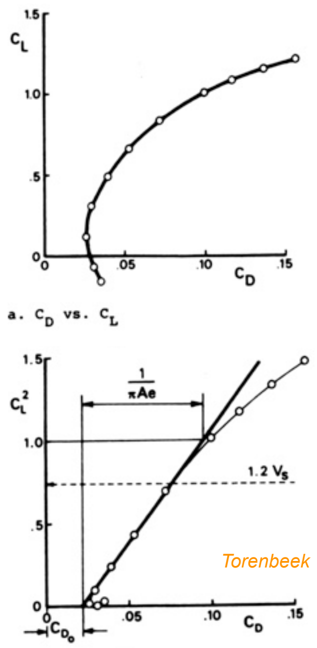
While the relationship between lift and drag is quite complicated in detail, a simple approximation is usually adequate over the operational range of C_L:

$$D = D_0 + D_i \qquad \text{or} \qquad C_D = C_{D_0} + C_{D_i} = C_{D_0} + \frac{C_L^2}{\pi A\, e}$$

where $C_{D,0}$ is called the 'zero-lift drag coefficient' (but strictly, isn't) and e is called the 'aircraft (or Oswald), efficiency factor' (similar to but usually smaller than the wing's span efficiency factor u).

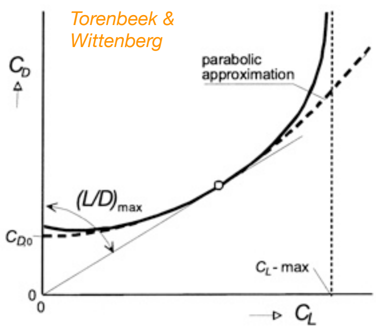
Strictly speaking, both $C_{D,0}$ and e are parameters in a simple parabolic approximation to the true drag polar.

Alternative nomenclature: $C_D = C_{D_0} + K C_L^2$ i.e. $K \equiv \dfrac{1}{\pi A\, e}$



a. C_D vs. C_L

b. C_D vs. C_L^2



The drag polar (and $C_{D,0}$, e, K) may change during flight since values depend both on Mach number and aircraft configuration (landing gear up/down, flaps deployed/retracted).

Next we look at $(L/D)_{\rm max}$ a.k.a. $(L/D)^*$.

Representative values:

	C_{D_0}	e
high-subsonic jet aircraft	.014 - .020	.75 - .85*
large turbopropeller aircraft	.018 - .024	.80 - .85
twin-engine piston aircraft	.022 - .028	.75 - .80
small single engine aircraft		
retractable gear	.020 - .030	.75 - .80
fixed gear	.025 - .040	.65 - .75

Torenbeek

AERODYNAMICS

Relations derived from drag polar: e.g $(C_L/C_D)^*$

A single figure of merit for aerodynamic properties is the lift:drag ratio $\frac{L}{D} = \frac{C_L}{C_D}$

The maximum value of this ratio is variously called $(L/D)_{max}$ or $(L/D)^*$ equivalently $(C_L/C_D)_{max}$ or $(C_L/C_D)^*$.

To find C_L for this maximum value we could take $$\frac{\mathrm{d}(C_L/C_D)}{\mathrm{d}C_L} = 0$$

But it is simpler to equivalently find $$\frac{\mathrm{d}(C_D/C_L)}{\mathrm{d}C_L} = 0$$

$$\frac{C_D}{C_L} = \frac{C_{D_0} + KC_L^2}{C_L} = \frac{C_{D_0}}{C_L} + KC_L \qquad \frac{\mathrm{d}(C_D/C_L)}{\mathrm{d}C_L} = -\frac{C_{D_0}}{C_L^2} + K = 0$$

Calling the value of C_L which achieves this turning point C_L*, $$C_L^* = \sqrt{\frac{C_{D_0}}{K}}$$

NOTE that C_L* (C_L value at the best ratio of lift to drag) is NOT THE SAME as C_{Lmax} (the highest value of C_L); typically C_L* is substantially lower than C_{Lmax}.

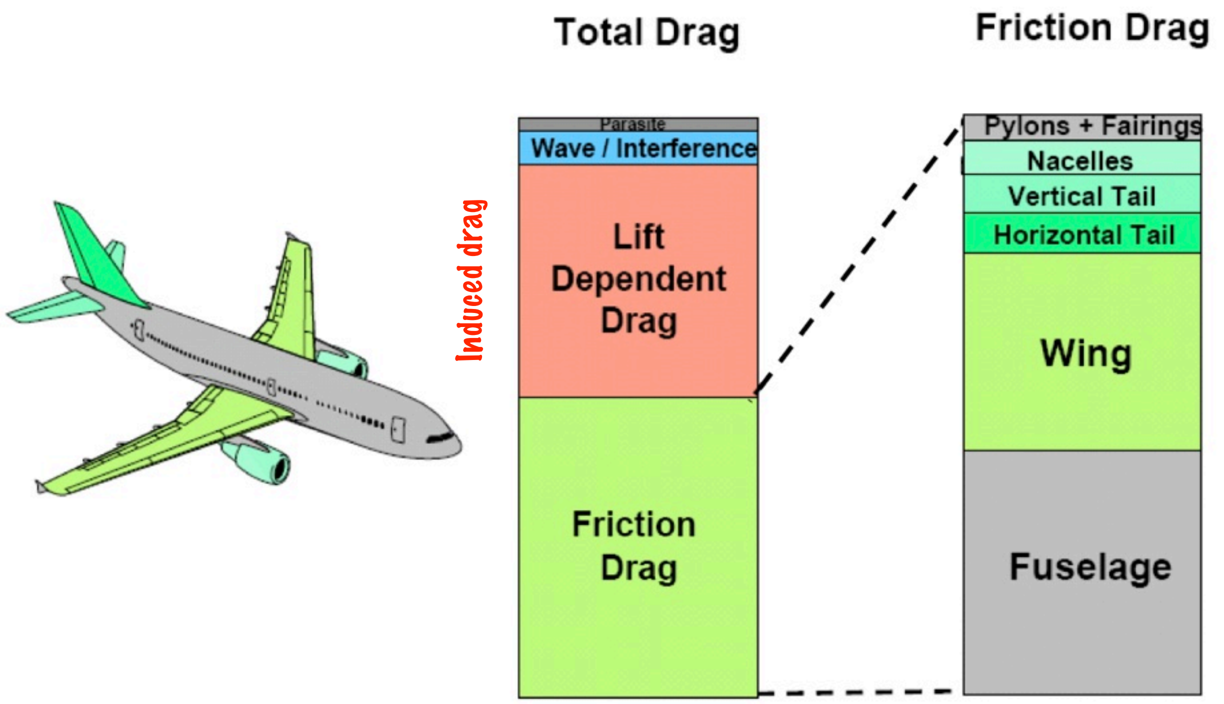
Corresponding value of C_D, $$C_D^* = 2C_{D_0}$$

And finally, the 'aerodynamic efficiency' $$C_L^*/C_D^* = (C_L/C_D)^* = \sqrt{\frac{C_{D_0}}{K}}\frac{1}{2C_{D_0}} = \frac{1}{\sqrt{4C_{D_0}K}}$$

This is just one optimal aerodynamic ratio. We'll see soon that there are others.

AERODYNAMICS

Typical contributions to drag for subsonic transport in cruise



G. Schrauf, AIAA 2008

Relationships between drag parameters

$$\left(\frac{C_L}{C_D}\right)_{\max} \equiv \left(\frac{C_L}{C_D}\right)^* = \frac{1}{\sqrt{4C_{D,0}K}} = \sqrt{\frac{\pi A e}{4C_{D,0}}}$$

Obviously A, e, and $C_{D,0}$ are important. But there other ways of looking at this: $= \sqrt{\frac{\pi b^2 e}{4C_{D,0}S}}$

Now suppose $C_{D,0}S \equiv C_{D,\text{wet}}S_{\text{wet}}$ where S_{wet} is 'wetted' surface area and $C_{D,\text{wet}}$ is a 'skin friction + form drag' coefficient.

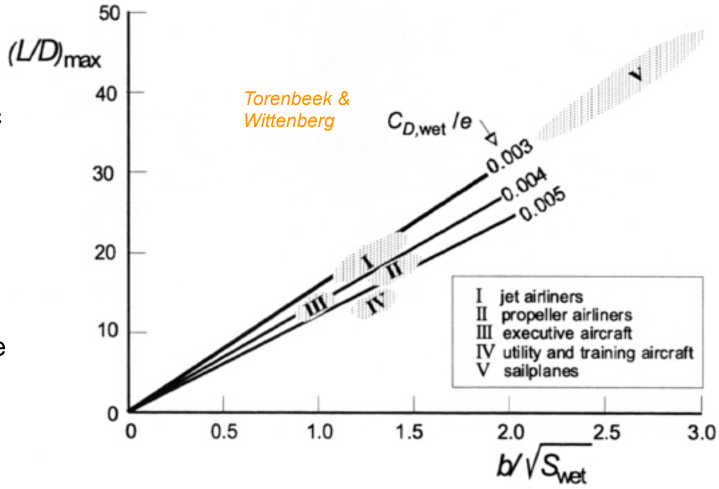
(It is reasonable to relate the zero-lift drag to area related directly to the total area with boundary layer drag.)

Then $$\left(\frac{L}{D}\right)_{\max} = \frac{1}{2}\sqrt{\frac{\pi e}{C_{D,\text{wet}}}}\frac{b}{\sqrt{S_{\text{wet}}}} \equiv \left(\frac{L}{D}\right)^* = \left(\frac{C_L}{C_D}\right)^*$$

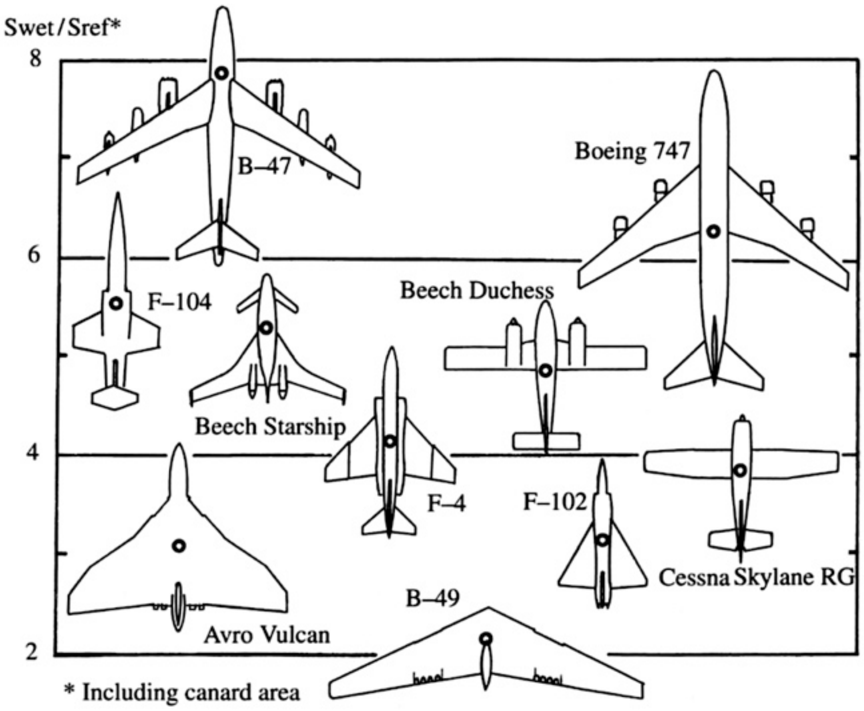
$C_{D,\text{wet}}/e$ is approximately constant within each category of aircraft since it relates to aerodynamic layout, surface fit and finish, which are broadly similar within categories.

Then $(L/D)_{\max}$ should be approximately linearly related to $b/\sqrt{S_{\text{wet}}}$ in each category.

These ideas do a reasonable job of correlating the observed aerodynamic efficiencies for various aircraft categories.



Wetted area S_{wet} vs wing reference area S or S_{ref}



Raymer

This plot gives representative approximations only.

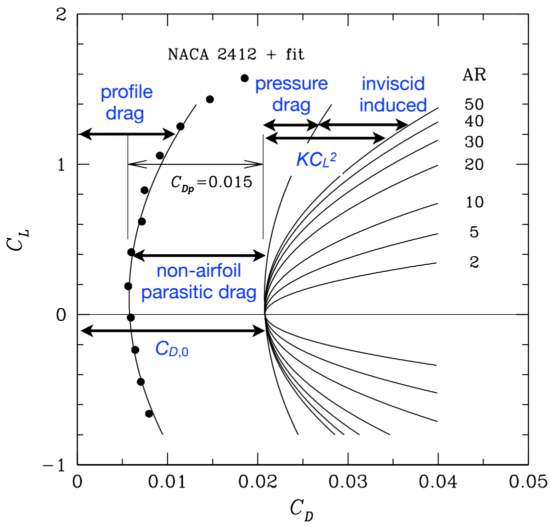
Different drag sources dominate at different speeds

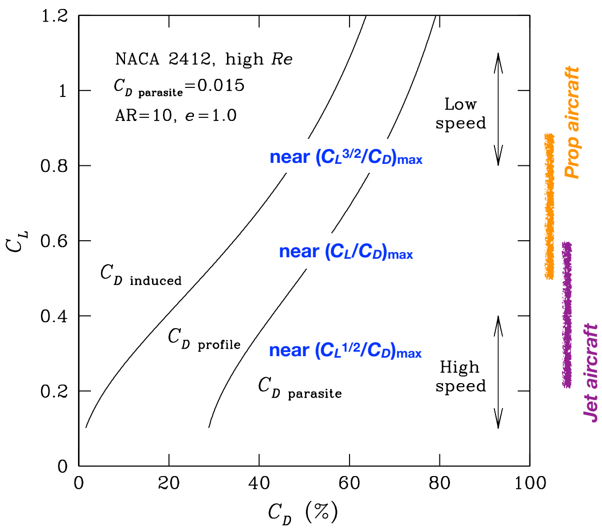
Recall $D = D_p + D_{\text{par}} + D_i$ with $C_D = \dfrac{D}{q_\infty S} = \dfrac{D}{\frac{1}{2}\rho V_\infty^2 S} = C_{D_p} + C_{D_{\text{par}}} + \dfrac{C_L^2}{\pi A\, e}$

(Airfoil profile; Parasitic drag; Induced drag)

and for level flight, $L = W$ so that $C_L = \dfrac{L}{q_\infty S} = \dfrac{W}{\frac{1}{2}\rho V_\infty^2 S}$ slow speed, high C_L; high speed, low C_L

So the relative importance of the induced drag term D_i varies with flight speed (i.e. $1/C_L^{1/2}$).





This makes different sources of drag more, or less, relatively important at different flight phases and for different aircraft types.