1

Preliminary sizing/weight estimation

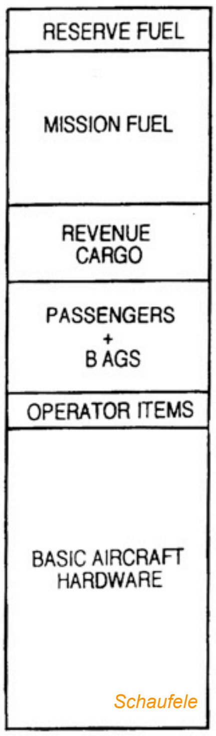
Recommended reading:

Nicolai & Carichner: Chapter 5, Appendix I
Torenbeek: Chapter 5
Raymer: Chapter 3

2

Nomenclature for weights/masses

Various names/acronyms exist for aircraft weights in different loading states (one set per text?).



WEIGHT BUILDUP

— MTOW, maximum take-off (gross) weight
a.k.a W_0
MTOM in UK texts

— MLW, maximum landing weight
— NLW, normal landing weight
— MZFW, maximum zero-fuel weight
a.k.a W_Z

MFW, maximum fuel weight (based on tank volume and fuel density) is not shown on this diagram.

— OWE, operator's weight empty OEM W_E
— MWE, manufacturer's weight empty

Typically, US texts refer just to 'weights' (since a pound mass and a pound force have the same numerical value) whereas texts based in SI (where mass is measured in kilogram and force is measured in Newton) refer to masses as well as weights.

Where US texts (e.g. Raymer) give conversions to SI 'weights' they are often given in kilogram (i.e. mass) units instead of Newton (weight).

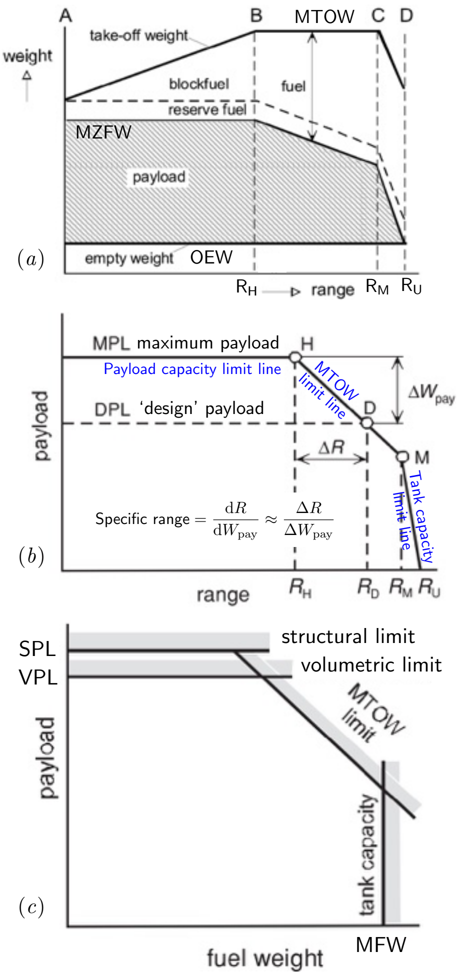
We will have to work around this (since we will use some US texts as reference material) but we should quote 'weights' in Newton.

Whatever the nomenclature used, from the point of view of aircraft design there are often just four weights (or masses) of interest:

$$W_0 = W_{\text{empty}} + W_{\text{payload}} + W_{\text{fuel}}$$

1. W_{fuel} could be thought of as energy weight.
2. $W_{\text{fuel}} = W_0 - W_Z$, if fuel is burned off.

— Note that e.g. it is sometimes unclear if operator items are included in empty weights quoted in manufacturer's data.



Torenbeek (2009, 2013)

Transport aircraft Payload–Range

1. All aircraft have a maximum takeoff weight and (typically) a maximum fuel tank capacity, but can operate at lower weight and less initial fuel. The payload-range diagram shows the interplay between payload, fuel weight and range.
2. MTOW = maximum certified takeoff weight.
3. OEW = Operator's empty weight.
4. SPL = MPL, maximum payload limited by structural strength.
5. SPL + OEW = MZFW.
6. VPL = space/volume limited payload (e.g. at maximum allowed single-class seating capacity and baggage). VPL <= SPL.
7. R_H = 'harmonic' range, the achievable range for takeoff at MTOW and MPL. Fuel is less than tank capacity.
8. R_D = 'design' range, the achievable range at design payload (e.g. multi-class seating capacity and baggage). Payload is reduced but range increases because allowable fuel increases.
9. Generally, the top-level requirements specify either R_H or R_D.
10. R_M = 'maximum' range achievable when departing at MTOW and maximum fuel capacity.
11. R_U = 'ultimate' or ferry range achievable with maximum fuel capacity and no payload.
12. Maximum fuel weight, MFW, depends on tank capacity.

FYI: maximum and normal landing weights

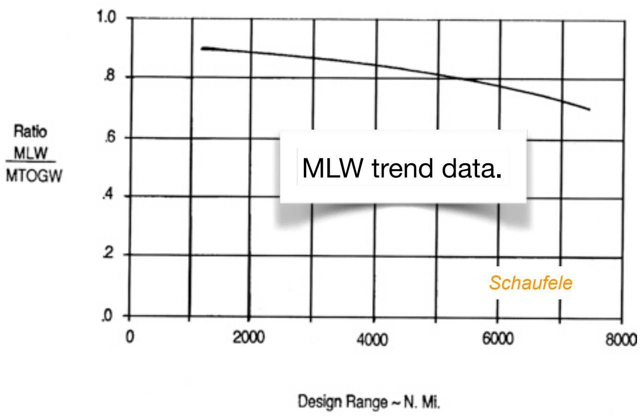
Structural design requirements for landing are normally to be met at *maximum landing weight* (MLW). For safety purposes this is different from the normal landing weight at the end of a long flight, and/or MZFW.

For small aircraft, MLW is typically chosen to be the same as MTOW.

However, for larger, long-range aircraft where full-mission fuel is a large proportion of MTOW, it is typical to use a smaller MLW, as shown.

This minimizes the structural weight impact of designing all the structure to withstand the loads associated with landing at MTOW.

Fuel dump systems are used to jettison fuel in an emergency following a MTOW takeoff, reducing weight to the design MLW prior to landing.




Fuel jettison


B747 at touchdown

For operational requirements, landing specifications are usually given at the *normal landing weight* (NLW).
NLW = OWE + full payload weight + reserve fuel weight.

5

Overview of initial weight estimation

'Unity equation'

$W_0 = W_{\text{payload}} + W_{\text{empty}} + W_{\text{fuel}}$ or $W_0 = W_{\text{p}} + W_{\text{e}} + W_{\text{f}}$ or $1 = W_{\text{p}}/W_0 + W_{\text{e}}/W_0 + W_{\text{f}}/W_0$

Rearrange to put unknown parts on one side and known/specified parts on the other.

(Note that the known parts might e.g. include weight of already chosen engines, see Torenbeek 5.5.2. In the following we assume that only the payload is specified. Straightforward – with care! – to generalise to other cases.)

$W_0 - W_{\text{f}} - W_{\text{e}} = W_{\text{p}}$ or $W_0(1 - W_{\text{f}}/W_0 - W_{\text{e}}/W_0) = W_{\text{p}}$

$$\boxed{W_0 = \frac{W_{\text{p}}}{1 - W_{\text{f}}/W_0 - W_{\text{e}}/W_0}}$$

Two typical approximate approaches for initial estimates.

Describes energy requirements. Estimated from mission profile, broad-brush aero and propulsion characteristics.

Describes empty structural (+ propulsion system, avionics, ...) weight. Estimated from correlations (i.e. curve fits based on historical data for class of aircraft) or reasonable alternative – anything is OK provided you can justify it!

1. NB: $(W_{\text{f}}/W_0 + W_{\text{e}}/W_0) < 1$, otherwise MTOW is negative. Not possible!
2. The ratio $1/(1 - W_{\text{f}}/W_0 - W_{\text{e}}/W_0)$ can be thought of as a multiplier of the payload weight. For large passenger transport aircraft, W_f/W_0 and W_e/W_0 are both of order 0.4, so the multiplier is order 5! In other words, MTOW is rather sensitive to any fixed (or extra dead, unanticipated) weight.
3. W_f/W_0 usually estimated with Breguet-based methods in the case of fuel-burning aircraft (the norm).
4. *Adopting any correlation data at this stage implies that you can build an aircraft to match!*

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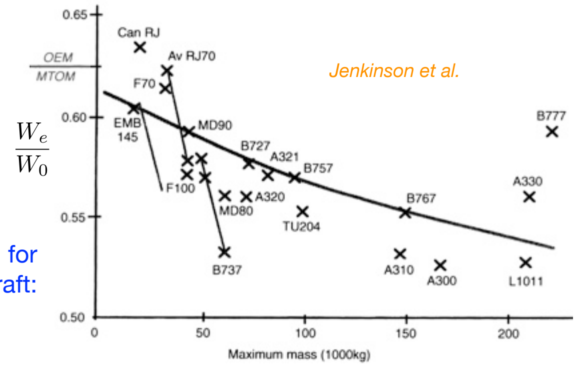
$$W_0 = \frac{W_{\text{p}}}{1 - W_{\text{f}}/W_0 - W_{\text{e}}/W_0}$$

W_e/W_0 correlations: the facts

1. The correlations are ONLY CURVE FITS typically based on a rather small sample for a specific aircraft category and hence have an implied (though usually not stated) uncertainty. They are not very reliable, and are suitable for first-cut estimates only.

Example fitted curve / correlation for W_e/W_0 of civil jet transport aircraft:



2. W_e/W_0 fraction correlations typically are also weakly decreasing functions of W_0. The corresponding implication is that aircraft get structurally more efficient the bigger they are – reflecting that it gets easier to find useable contiguous volumes in larger aircraft as relative size of discrete components (rivets, connectors, seats – passengers! – etc.) falls.
3. Most W_e/W_0 fraction correlations given in texts are basically of power-law form, but somehow each text seems to have its own way of describing this simple form! Be prepared to convert and compare.
4. Because of this power-law form, most of the constants (but not exponents) are of dimensional form and one must pay careful attention to this fact, and that their numerical values will change with change unit system used (many texts use Imperial units). Again, be prepared to convert and compare.
5. Correspondingly, the correlations are particular to the materials/technology used in the structures on which they were based and 'adjusting' them for new technologies (e.g. fibre composites instead of aluminium alloy) introduces additional uncertainty.
6. Be careful to check what components are included in the correlations (do they include internal fit-out? Fuel system?). If you are going to break some 'known weight' (e.g. specific engine choice) out of the correlation used, be prepared to adjust.

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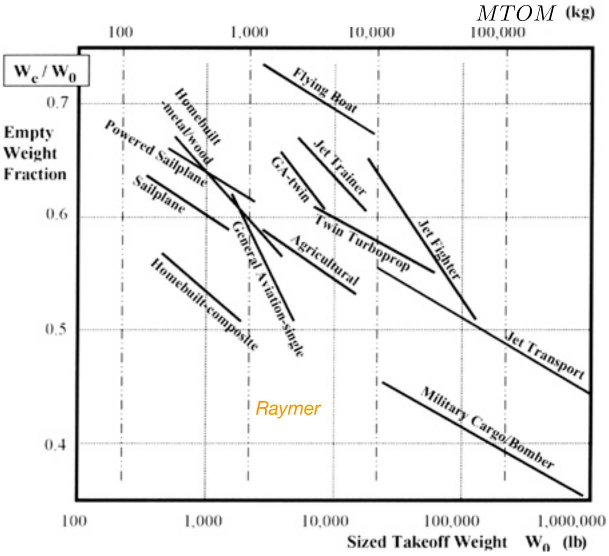
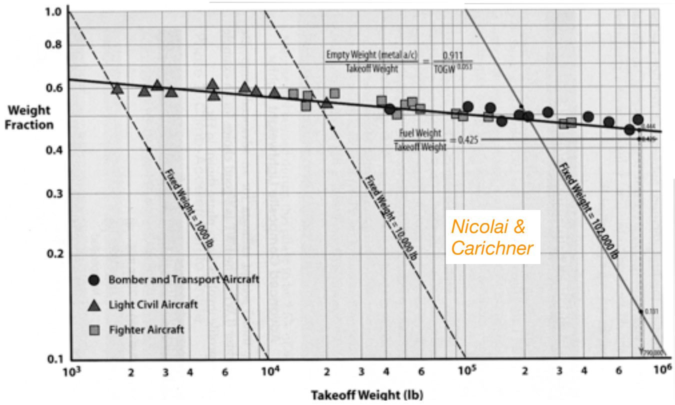
Example weight fraction correlations



(Raymer's) Equivalent curve fits for W_e/W_0:

$W_e/W_0 = A \times MTOM^C$	{A-metric}	C
Sailplane—unpowered	{0.83}	−0.05
Sailplane—powered	{0.88}	−0.05
Homebuilt—metal/wood	{1.11}	−0.09
Homebuilt—composite	{1.07}	−0.09
General aviation—single engine	{2.05}	−0.18
General aviation—twin engine	{1.4}	−0.10
Agricultural aircraft	{0.72}	−0.03
Twin turboprop	{0.92}	−0.05
Flying boat	{1.05}	−0.05
Jet trainer	{1.47}	−0.10
Jet fighter	{2.11}	−0.13
Military cargo/bomber	{0.88}	−0.07
Jet transport	{0.97}	−0.06

NB: *A* are dimensional constants. Values above for {A} assume *MTOM* is in kg.



Most aircraft design texts have some equivalent set of correlations (another is shown at right). It is good practice to check the effect of choosing different correlations on initial weight estimate.

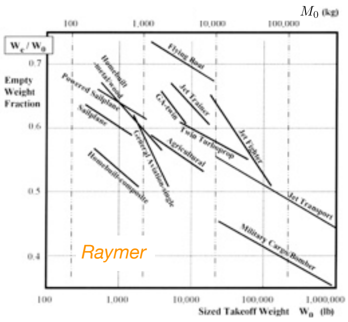
Because the constants (not exponents) in such fits typically have dimensions, care must be used when adopting curve fits from US texts.

8

Various forms of power-law type empty weight correlations

1. Raymer gives correlations in the form

$$M_e/M_0 \equiv W_e/W_0 = AM_0^B$$



2. Nicolai & Carichner give correlations in the form

$$M_e = CM_0^D$$

but it is easy to see that

$$M_e/M_0 = CM_0^{D-1}$$

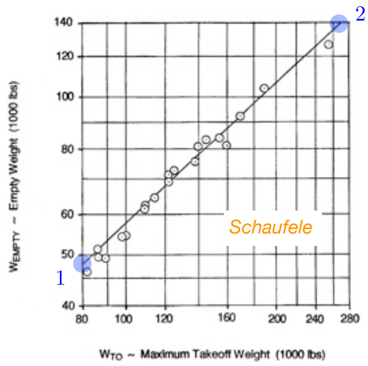
and hence that $C \equiv A,\ D - 1 \equiv B$

3. Roskam gives the form $M_e = 10^{(\log M_0 - G)/H}$

but after some rearrangement we find

$$10^{-G/H} \equiv A,\ (1 - G)/G \equiv B$$

4. Schaufele provides his correlations in graphical power-law form only



from which Nicolai & Carichner's form is obtained:

$$D = \frac{\log(M_{e_2} - M_{e_1})}{\log(M_{0_2} - M_{0_1})} \qquad C = 0.5\left(\frac{M_{e_1}}{M_{0_1}^D} + \frac{M_{e_2}}{M_{0_2}^D}\right)$$

5. Finally we must be ready to convert units. E.g. if *C* and *D* are found in Imperial (lbm, lbf) units then for an SI conversion the exponent *D* remains the same but

$$C_{\mathrm{SI}} \equiv 2.205^{D-1} C_{\mathrm{IMP}}$$

where 1 kg = 2.205 lb.

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$$W_0 = \frac{W_p}{1 - W_f/W_0 - W_e/W_0}$$

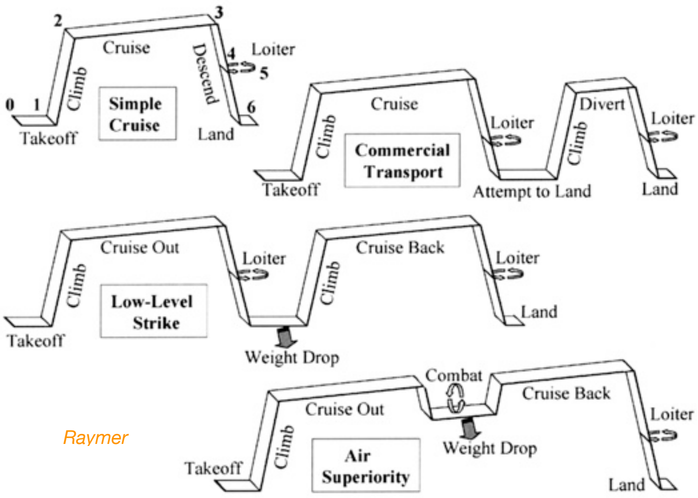
W_f/W_0 estimation: the big picture

Energy source		Conversion efficiencies		Propulsive energy requirement		Heating of fluid medium
Hydrocarbon fuel, hydrogen fuel, battery, fat or sugar, rubber band, …	→	Gas turbine, reciprocating engine, speed controller+electric motor, muscle, propeller, …	→	Drag force x distance	→	Viscous dissipation of kinetic energy
		Irreversible heating = loss				**Irreversible heating = loss**

1. Fuel weight fraction estimation is essentially energy use estimation. Usually the dominant component of energy use is for steady level flight, and the underlying equation is $T = D = W/(L/D)$. If W is constant, then propulsive energy required is then TR (thrust x range), or alternatively PE (power x endurance) = TV x E. Note that L/D depends only on aerodynamics (not on aircraft structure).
2. If W reduces with time (fuel is burned) then the Breguet equation or some related equation is used. Different equations may be used for different *mission segments* (e.g. range vs endurance).
3. For mission segments that are too complicated for simple analysis, historical estimates for fractional energy use are sometimes introduced in first-pass estimation.
4. One may/may not have to include various energy conversion efficiencies in the calculations in order to get back to the amount of chemical/electrical/other energy that must be consumed to get a required amount of propulsive energy. (For jet powered aircraft, propulsive efficiency is usually buried in thrust-specific fuel consumption value c_t, while for propeller aircraft the propulsive or propeller efficiency is usually broken out as a separate value and not buried in the power-specific fuel consumption value c_p.)

10

Mission analysis for fuel-burning aircraft



Raymer

Fig. 3.2 Typical mission profiles for sizing.

Typically, aircraft requirements are given in terms of a mission profile and performance requirements during various mission phases or segments:

<u>Break the required mission up into numbered segments. 0-1 is takeoff, 1-2 is climb, etc.</u>

Two typical longer segments are 'cruise' and 'loiter'.

Cruise ≡ range,
Loiter ≡ endurance

<u>It is standard practice to show such a mission profile in design reports.</u>

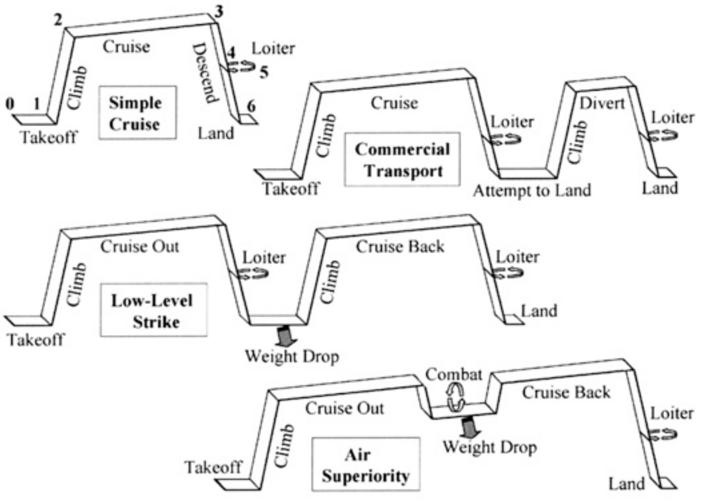
Supposing there are n flight segments and fuel is burned in each segment. Then

$$\frac{W_n}{W_0} = \frac{W_n}{W_{n-1}} \times \frac{W_n}{W_{n-1}} \cdots \frac{W_i}{W_{i-1}} \cdots \frac{W_2}{W_1} \times \frac{W_1}{W_0} \equiv \prod_{i=1}^{n} \frac{W_i}{W_{i-1}}$$

Since the difference between W_n and W_0 must be the amount of fuel used, W_{fuel},
$$\frac{W_{\text{fuel}}}{W_0} = \left(1 - \frac{W_n}{W_0}\right)$$

Often a reserve/safety margin is added (if none given), e.g = 5+1% = 6%; SF = 1.06
$$\frac{W_{\text{fuel}}}{W_0} = \text{SF}\left(1 - \frac{W_n}{W_0}\right)$$

Mission analysis for fuel-burning aircraft



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Typically, aircraft requirements are given in terms of a mission profile and performance requirements during various mission phases or segments:

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Two typical longer segments are 'cruise' and 'loiter'.

Cruise ≡ range,
Loiter ≡ endurance

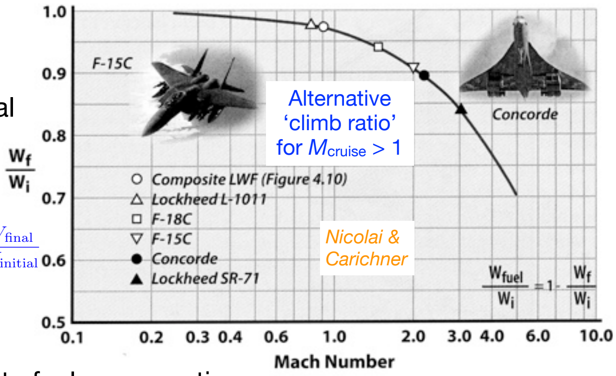
(More detailed correlation of historical data for climb–acceleration phase.)

For the short segments, we can often use statistics / historical values to tell us W_i/W_{i-1}, corresponding to fuel usage for segment i. (Good enough for first-pass estimates.)

Mission segment	(W_i/W_{i-1})
Warmup and takeoff	0.970
Climb	0.985
Landing	0.995

Raymer

$$\frac{W_i}{W_{i-1}} \equiv \frac{W_f}{W_i} \equiv \frac{W_{\text{final}}}{W_{\text{initial}}}$$



<u>For long segments, we use Breguet-type equations to estimate fuel consumption.</u>

Recall: Breguet-type fuel consumption analysis

The two main tools here are the Breguet-type range and endurance equations. Warning: be very careful with dimensional consistency when adopting equations from (US-based) texts.

Type | **Equation** | **Optimal flight strategy** | **Segment weight fraction**

<u>Range, *R*</u>

Jet $R = \frac{1}{gc_t} V \frac{L}{D} \ln \frac{W_{i-1}}{W_i}$

Maximized at high altitude (high $V \Rightarrow$ low ρ).
Fly at $1.316V^*$, $0.577C_L^*$, $0.866(C_L/C_D)^*$.
<u>BUT if $M_{DD} < 1.316V^*$, fly at M_{DD}, C_L^*, $(C_L/C_D)^*$.</u>

$$\boxed{\frac{W_i}{W_{i-1}} = \exp \frac{-Rgc_t}{V(L/D)}}$$

M_{DD} = drag divergence Mach number.

Prop $R = \frac{\eta_{\text{pr}}}{gc_p} \frac{L}{D} \ln \frac{W_{i-1}}{W_i}$

Independent of altitude.
Fly at V^*, C_L^*, $(C_L/C_D)^*$.

$$\boxed{\frac{W_i}{W_{i-1}} = \exp \frac{-Rgc_p}{\eta_{\text{pr}}(L/D)}}$$

<u>Endurance, *E*</u>

Jet $E = \frac{1}{gc_t} \frac{L}{D} \ln \frac{W_{i-1}}{W_i}$

Independent of altitude.
Fly at V^*, C_L^*, $(C_L/C_D)^*$.

$$\boxed{\frac{W_i}{W_{i-1}} = \exp \frac{-Egc_t}{(L/D)}}$$

Prop $E = \frac{\eta_{\text{pr}}}{gc_p} \frac{1}{V} \frac{L}{D} \ln \frac{W_{i-1}}{W_i}$

Maximized at low altitude (low $V \Rightarrow$ high ρ).
Fly at $0.760V^*$, $1.732C_L^*$, $0.866(C_L/C_D)^*$.
V^* reduces as W falls, h fixed near SL.

$$\boxed{\frac{W_i}{W_{i-1}} = \exp \frac{-Egc_pV}{\eta_{\text{pr}}(L/D)}}$$

Function	***Dimensionless***	**V/V^*, at max**	**$(L/D)/(L/D)^*$, at max**	**C_L/C_L^*, at max**
$L/(DV)$	$C_L^{3/2}/C_D$	$(1/3)^{1/4} = 0.760$	$(3/4)^{1/2} = 0.866$	$3^{1/2} = 1.732$
L/D	C_L/C_D	1	1	1
$(VL)/D$	$C_L^{1/2}/C_D$	$(3)^{1/4} = 1.316$	$(3/4)^{1/2} = 0.866$	$(1/3)^{1/2} = 0.577$

Drag rise in transonic flight

Torenbeek & Wittenberg

Airfoil lift and drag with M, fixed α

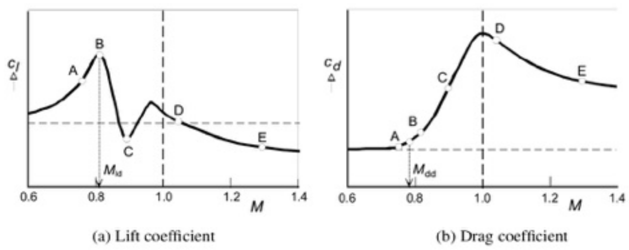


Figure 9.21 Aerodynamic coefficients in the transonic speed range for the wing section in Figure 9.20.

Wing lift and drag with M

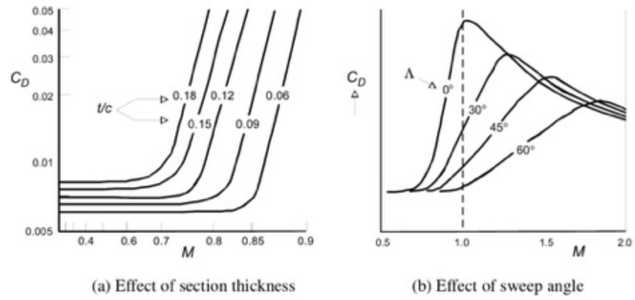


Figure 9.22 Influence of the wing shape on the drag at transonic speed.

Whole aircraft lift and drag with M, fixed sweep

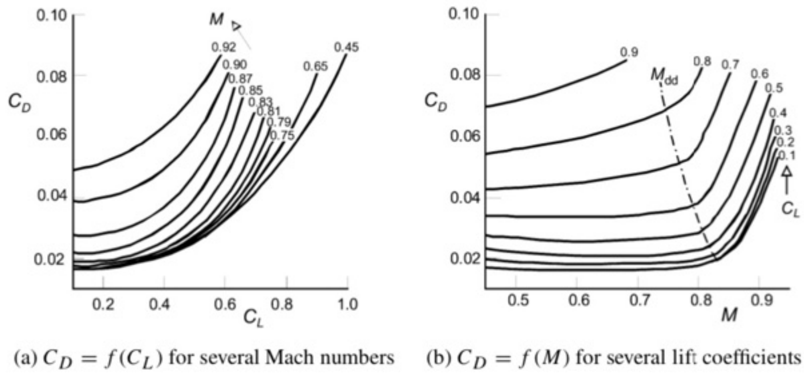


Figure 9.37 Drag coefficient of a jet airliner at subsonic and transonic speeds.

$$C_L = C_L(\alpha, M), \quad C_D = C_D(\alpha, M) \quad \rightarrow \quad C_D = C_D(C_L, M)$$

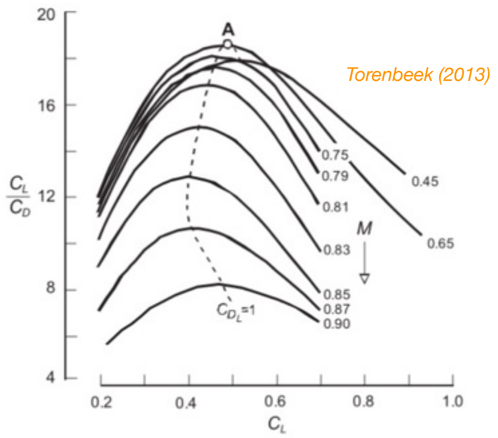


Figure 12.1 Aerodynamic efficiency of a transonic airliner

Optimal flight strategies for jet aircraft range

For jet aircraft range, the product

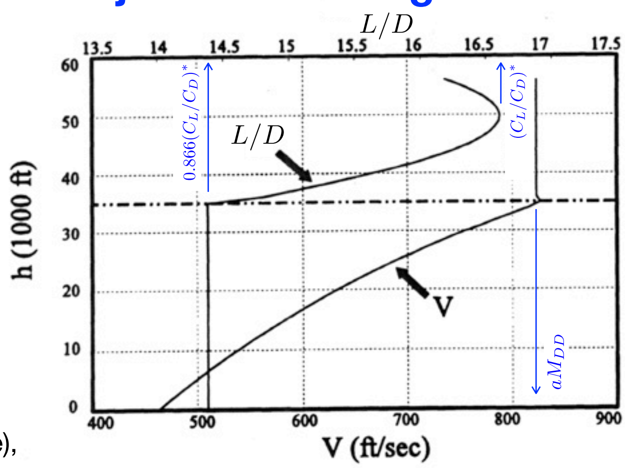
$$V\frac{L}{D} = V\frac{C_L}{C_D} = a\,M\frac{C_L}{C_D}$$
$$= \sqrt{\frac{2}{\rho}\frac{W}{S}\frac{1}{C_L}}\,\frac{C_L}{C_D} = \sqrt{\frac{2}{\rho}\frac{W}{S}}\,\frac{C_L^{1/2}}{C_D}$$

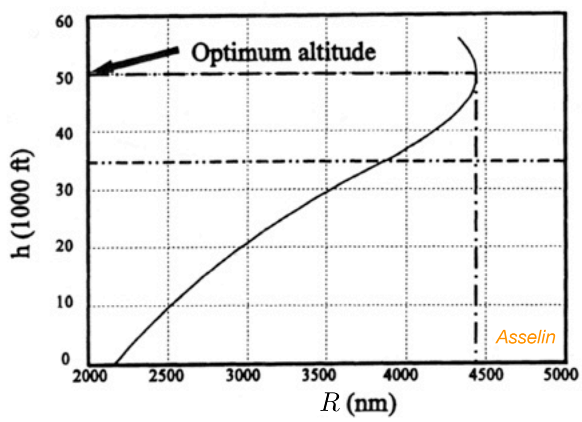
can, if speed is not limited, be maximised at a fixed wing loading by flying at $(C_L^{1/2}/C_D)_{max}$ (where $C_L/C_D \approx 0.866\,(C_L/C_D)^*$) and raising h i.e. decreasing ρ (indefinitely).

<u>If, however, the speed is fixed/limited</u> (say at M_{DD} which is the effective maximum for transonic cruise), then further improvement can only be obtained by increasing L/D. **In this case the range is approximately maximized at M_{DD}, and at $(C_L/C_D)^*$.**

These ideas form the basis for typical climb/speed strategies pursued in practice – see diagram to right from Asselin's *An Introduction to Aircraft Performance*.

See the more complete discussion in Torenbeek & Wittenberg § 9.10. A detailed explanation of their analysis is supplied in Torenbeek (2013) Chs 2 and 12. He also shows how to account for variation of the drag polar, and c_t with M and h.





15

Initial aerodynamic estimates – 1

To make further progress we need initial guess $(L/D)^*$ estimates. If we have reliable values for $C_{D,0}$ and K we can use these, but it is typical in initial work to use correlation-based values for similar aircraft types.

$$\left(\frac{C_L}{C_D}\right)_{\max} \equiv \left(\frac{C_L}{C_D}\right)^* = \frac{1}{\sqrt{4C_{D,0}K}} = \sqrt{\frac{\pi A e}{4C_{D,0}}}$$

Obviously A, e, and $C_{D,0}$ are important. But there other ways of looking at this: $= \sqrt{\dfrac{\pi b^2 e}{4C_{D,0}S}}$

Now suppose $C_{D,0}S \equiv C_{D,\mathrm{wet}}S_{\mathrm{wet}}$ where S_{wet} is 'wetted' surface area and $C_{D,\mathrm{wet}}$ is a 'skin friction + form drag' coefficient. C_{fe} is sometimes used as a synonym for $C_{D,\mathrm{wet}}$.

(It is reasonable to relate the zero-lift drag to area related directly to the total area with boundary layer drag.)

Then $$\boxed{\left(\frac{L}{D}\right)_{\max} = \frac{1}{2}\sqrt{\frac{\pi e}{C_{D,\mathrm{wet}}}}\frac{b}{\sqrt{S_{\mathrm{wet}}}}}$$

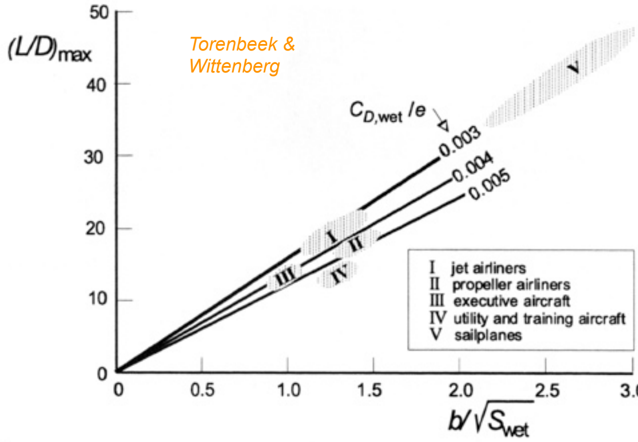
$$\frac{b^2}{S_{\mathrm{wet}}} = \frac{b^2}{S}\frac{S}{S_{\mathrm{wet}}} = A\frac{S}{S_{\mathrm{wet}}}$$ is called the 'wetted aspect ratio': a geometric property which tends to fall in a small band of values for different aircraft classes.

$C_{D,\mathrm{wet}}/e$ is approximately constant within each category of aircraft since it relates to aerodynamic layout, surface fit and finish, which are broadly similar within categories.

Then $(L/D)_{\max}$ should be approximately linearly related to $b/\sqrt{S_{\mathrm{wet}}}$ in each category.

These ideas do a reasonable job of correlating the observed aerodynamic efficiencies for various aircraft categories.

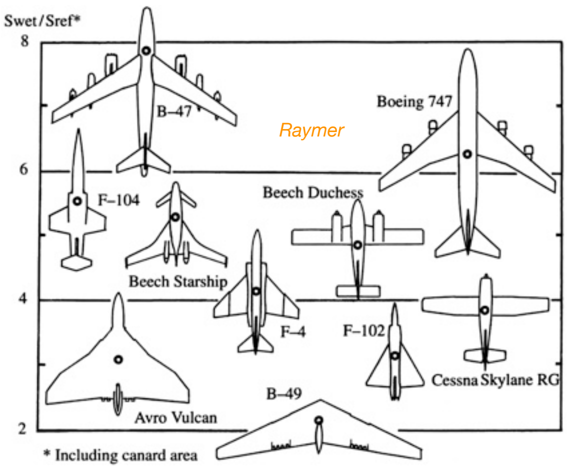
Note: it is important that as designers we ultimately come back and check our drag polar estimates.

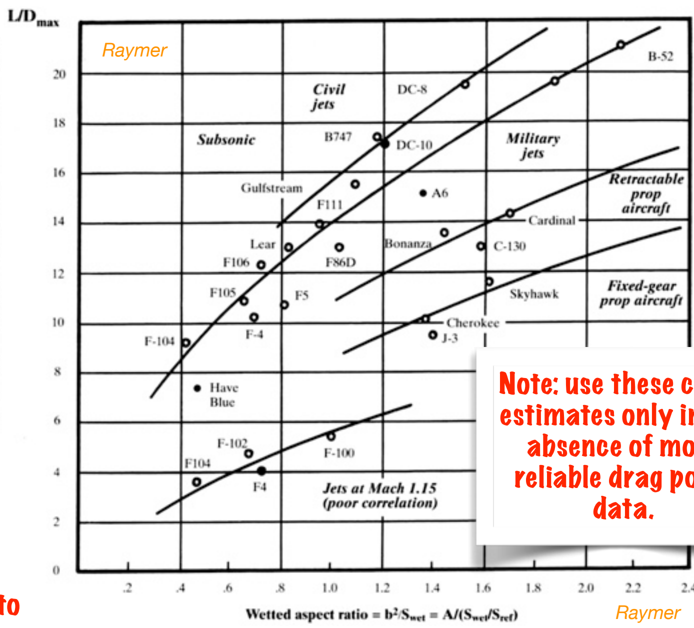


16

Initial aerodynamic estimates – 2

Alternatively, Raymer's approximate method employs a semi-guessed estimate of the ratio of aircraft 'wetted area' to (wing) reference area, i.e. $S_{\mathrm{wet}}/S_{\mathrm{ref}}$ and a 'wetted aspect ratio' $b^2/S_{\mathrm{wet}} = A/(S_{\mathrm{wet}}/S_{\mathrm{ref}})$. It is really just the same as Torenbeek & Wittenberg's method, but with some 'real data' added.





Note: use these crude estimates only in the absence of more reliable drag polar data.

Note: in order to use these methods, we have to have some idea of the aircraft layout.

Other texts, e.g. Nicolai & Carichner, supply equivalent first-pass data; it is good practice to cross-check.

	Cruise	Loiter
Jet	$0.866\ L/D_{\max}$	$L/D_{\max}$
Prop	$L/D_{\max}$	$0.866\ L/D_{\max}$

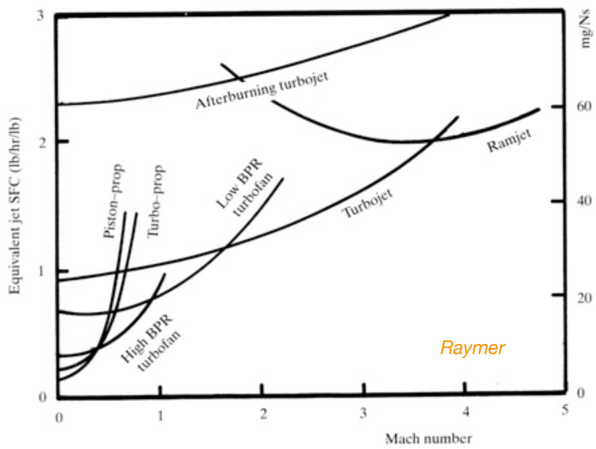
Finally, we don't use $L/D_{\max}$ in the Breguet equations, but the appropriately factored values we saw earlier.

Recall: thrust and power-specific fuel consumption

To make further progress with Breguet-type fuel use, we need to relate engine thrust or power to fuel consumption rates. While in later analyses it is proper to use information specific to the engine chosen, for initial weight estimation one uses typical values of thrust-power-specific fuel consumptions.

NB: the fuel consumption rates below are for typical aircraft hydrocarbon-based fuels only.

Representative fuel consumption rates:



c_t TSFC

Typical jet SFCs: 1/hr {mg/Ns}	Cruise	Loiter
Pure turbojet	0.9 {25.5}	0.8 {22.7}
Low-bypass turbofan	0.8 {22.7}	0.7 {19.8}
High-bypass turbofan	0.5 {14.1}	0.4 {11.3}

c_p PSFC

Propeller: $C = C$ power $V/\eta_p = C_{bhp}\ V/(550\eta_p)$

Typical C_{bhp}: lb/hr/bhp {mg/W-s}	Cruise	Loiter
Piston-prop (fixed pitch)	0.4 {.068}	0.5 {.085}
Piston-prop (variable pitch)	0.4 {.068}	0.5 {.085}
Turboprop	0.5 {.085}	0.6 {.101}

Raymer

The weight fraction β

Note for later reference in performance analysis: at any point in the flight the current weight $W_i = \beta_i W_0$. I.e.

$$\boxed{\beta_i \equiv \frac{W_i}{W_0} = \frac{W_i}{W_{i-1}} \cdots \frac{W_2}{W_1} \times \frac{W_1}{W_0}}$$

Initial weight estimate – methodology

Now we have a set of techniques that will enable us to estimate aircraft gross weight W_0.

However because the empty weight fraction W_e/W_0 correlation depends on the gross weight W_0, the method is iterative.

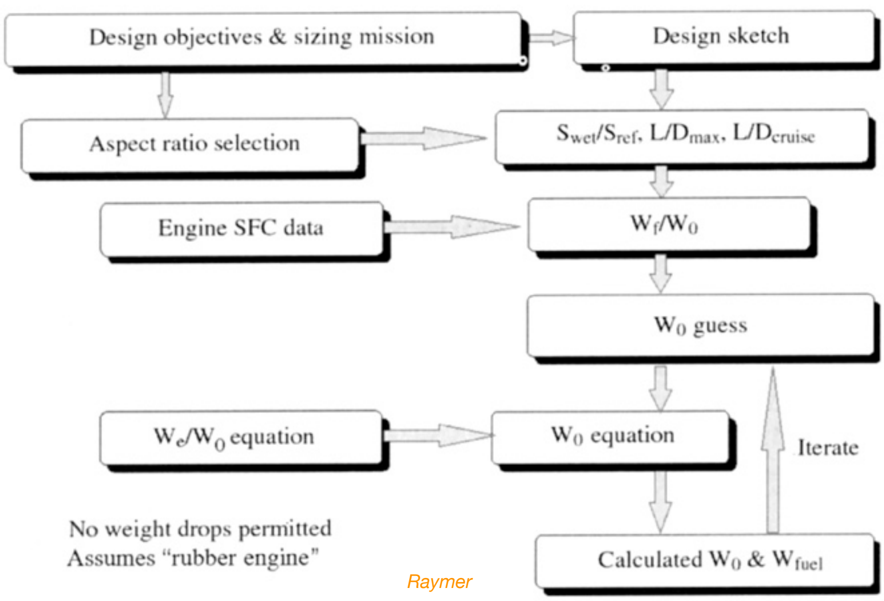
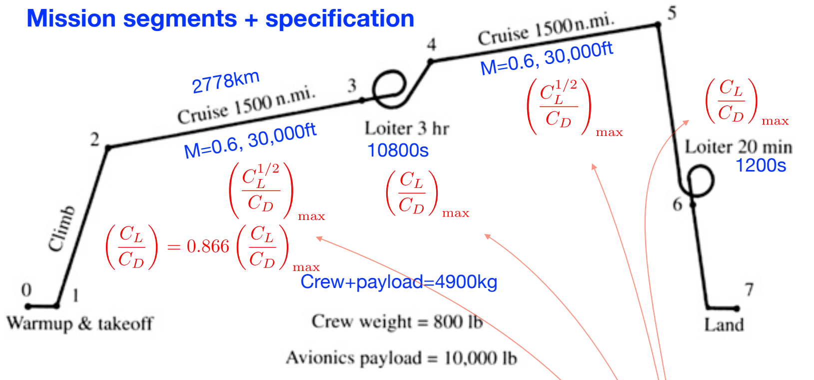


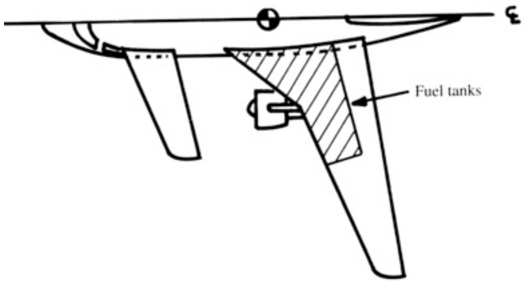
Fig. 3.7 First-order design method.

We will start with simple problems with fixed payload, and where cruise speeds and heights are given.

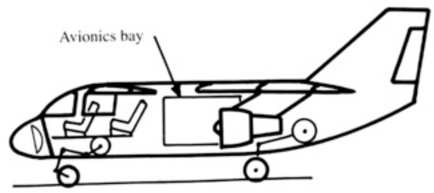
Example: Jet ASW aircraft — 1

Mission segments + specification





Configuration layout sketch: canard



Similar spec: Lockheed Viking S-3A

From the aircraft type (jet or prop), the drag polar locations at which cruise and loiter segments will be flown in order to maximise either range or endurance are fixed.

Layout-dependent values:

Wing aspect ratio $A \approx 10$

Including front wing (canard layout) $A \approx 7$

$S_{wet}/S_{ref} \approx 5.5$

Wetted aspect ratio 7/5.5=1.27

$L/D_{max} \approx 16$

Cruise: use $L/D = 0.866\ L/D_{max}$

Loiter: use $L/D = L/D_{max}$

Example: Jet ASW aircraft — 2

Note that we do not need to directly know the thrust.

Engines:- High-bypass turbofan

SFC(Thrust) $C_t = 14.1\times10^{-6}$ kg/Ns cruise

$C_t = 11.3\times10^{-6}$ kg/Ns loiter

Weight ratios by flight segment.

1 Warmup + takeoff $W_1/W_0 = 0.97$

2 Climb $W_2/W_1 = 0.985$

3 Cruise $R = 2.778\times10^6$ m

$C_t = 14.1\times10^{-6}$ kg/Ns

$V_a = 0.6\times303.2$ m/s $= 182$ m/s

$L/D = 0.866\times16 = 13.9$

From Breguet $W_3/W_2 = \exp\left(-\frac{R g C_t}{V_a (L/D)}\right)$

$= \exp\left(-\frac{2.778\times10^6\times9.8\times14.1\times10^{-6}}{182\times13.9}\right)$

$= \exp(-0.152) = 0.859$

4 Loiter $E = 10{,}800$ s

$C_t = 11.3\times10^{-6}$ kg/Ns

$L/D = 16$

From Endurance $W_4/W_3 = \exp\left(-\frac{E g C_t}{L/D}\right)$

$= \exp\left(-\frac{10800\times9.8\times11.3\times10^{-6}}{16}\right)$

$= \exp(-0.0748) = 0.928$

5 Cruise same as 3 $\frac{W_5}{W_4} = 0.859$

6 Loiter $E = 1200$ s

$W_6/W_5 = \exp(-0.00831) = 0.992$

7 Land $W_7/W_6 = 0.995$.

Overall mission weight ratio

$\frac{W_7}{W_0} = 0.97\times0.985\times0.859\times0.928\times0.859\times0.992\times0.995 = \underline{0.646}$.

$W_f/W_0 = 1.06(1-0.646) = 0.375$ (3/8)

Correlation for military cargo/bomber

$W_e/W_0 = 0.88\,W_0^{-0.07}$, W_0 in kg.

$W_0 = \frac{W_{crew} + W_{payload}}{1 - \frac{W_f}{W_0} - \frac{W_e}{W_0}} = \frac{4900}{1-0.375-0.88W_0^{-0.07}}$

Iterate by successive substitution

W_0 guess	W_e/W_0	W_0 calc
25000	0.4531	25539
25539	0.4325	25454
25454	0.4326	25467
25467	0.4326	25465 ✓ OK

Answer: estimate $W_0 = 25465$ kg

cf Lockheed S-3A: 23830 kg.

Performance-based fuel consumption analysis

1. To this point we have used a simplified method where historical estimates have been used for aircraft weight ratios at the beginning and end of relatively complex (but short) flight segments including take-off and landing and where fuel consumption is dominated by long cruise/loiter segments.
2. More generally (and accurately) we may calculate fuel consumption for the more complex segments (including manoeuvres) based on their duration (*d*) and fuel consumption rate.

From $\mathrm{d}W/\mathrm{d}t = -\mathrm{g}c_t T$ if using TSFC c_t, $\mathrm{d}W/\mathrm{d}t = -\mathrm{g}c_p P$ if using PSFC c_p, we get

$$W_{\mathrm{fuel},i} \approx \mathrm{g}\, c_t\, T_i\, d_i$$
$$W_{\mathrm{fuel},i} \approx \mathrm{g}\, c_p\, P_i\, d_i$$

3. From $W_{\mathrm{fuel},i} = W_{i-1} - W_i$ we have $\boxed{\dfrac{W_i}{W_{i-1}}} = 1 - \dfrac{W_{\mathrm{fuel},i}}{W_{i-1}} \approx 1 - \dfrac{\mathrm{g}c_t T_i d_i}{W_{i-1}} \boxed{\approx 1 - \mathrm{g}c_t d_i \left.\dfrac{T}{W}\right|_i}$ (for c_t case)
4. We can calculate *T*/*W* from the FPE*, rearranged as $\boxed{\dfrac{T}{W} = \dfrac{qS}{\beta W_0}\left[C_{D,0} + K\left(\dfrac{n\beta}{q}\dfrac{W_0}{S}\right)^2\right] + \dfrac{P_s}{V}}$

$$P_s = \frac{\mathrm{d}}{\mathrm{d}t}\left(h + \frac{V^2}{2g}\right)$$

5. However, the difficulty is that to complete the calculation we need β and W_0/S, which may not be known at this point. If that is the case we have to guess β and W_0/S, based on historical/reasonable estimates of β and past comparable designs for W_0/S. Then finally we will need to make (at least) another pass through the design loop when more accurate estimates of β and W_0/S are available.
6. Finally, even for the long cruise/loiter segments, it is typically best for accuracy to break each into a number of short segments and estimate the fuel use/weight ratio for each segment — which is just a generalisation of the above technique.
7. Because we may need to iterate the calculations as better estimates of W_0/S and β become available, it makes good sense to take a spreadsheet or computer-based approach to performing the overall fuel use calculations. (The same is true for detailed weight estimates.)

*FPE = 'Fundamental Performance Equation'.

Missions with weight drop — 1

1. The methodology as outlined so far assumes that all aircraft weight loss is due to fuel use. This may not be true in general as stores/payload may be jettisoned in flight.
2. Say that the payload weight W_p is broken into permanent and expendable parts: $W_p = W_{pp} + W_{pe}$.

$$W_0 = W_e + W_p + W_f = W_e + W_{pp} + W_{pe} + W_f$$

or (known vs unknown) $$W_e + W_{pp} = W_0 - W_f - W_{pe}$$

3. And that W_{pe} is jettisoned at the end of flight segment *j*: 0 — 1 — 2 — ... — *j* — ... *n*-1 — *n*

drop W_{pe}

Ratio of weight before and after weight drop.

4. Now: $W_e + W_{pp} = W_0 \times \dfrac{W_1}{W_0} \times \dfrac{W_2}{W_1} \times \cdots \times \dfrac{W_j}{W_{j-1}} \times \left(1 - \dfrac{W_{pe}}{W_j}\right) \times \dfrac{W_{j+1}}{W_j} \times \cdots \times \dfrac{W_n}{W_{n-1}}$

$$= W_0 \prod_1^j \left(1 - \frac{W_{pe}}{W_j}\right) \prod_j^n = W_0 - W_f - W_{pe}$$

Each of these computed by previous methods

Rearrange: $$W_f = W_0 - W_{pe} - W_0 \prod_1^j \left(1 - \frac{W_{pe}}{W_j}\right) \prod_j^n$$

$$= W_0 - W_{pe} - W_0 \prod_1^j \prod_j^n \left(1 - \frac{W_{pe}}{W_0 \prod_1^j}\right)$$

$$= W_0 - W_{pe} - W_0 \prod_1^n \left(1 - \frac{W_{pe}}{W_0 \prod_1^j}\right)$$

$$= W_0 - W_{pe} - W_0 \prod_1^n + W_{pe} \prod_j^n$$

or

$$\boxed{W_f = W_0 \left(1 - \prod_1^n\right) - W_{pe}\left(1 - \prod_j^n\right)}$$

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Missions with weight drop — 2

Recall $W_f = W_0\left(1 - \prod_1^n\right) - W_{pe}\left(1 - \prod_j^n\right)$

5. Substituting back:

$$\begin{aligned} W_0 &= W_e + W_{pp} + W_f + W_{pe} \\ &= W_e + W_{pp} + W_0\left(1 - \prod_1^n\right) - W_{pe}\left(1 - \prod_j^n\right) + W_{pe} \\ &= W_e + W_{pp} + W_0\left(1 - \prod_1^n\right) - W_{pe}\prod_j^n \end{aligned}$$

or

$$1 = \frac{W_e}{W_0} + \frac{W_{pp}}{W_0} + \left(1 - \prod_1^n\right) + \frac{W_{pe}}{W_0}\prod_j^n$$

$$0 = \frac{W_e}{W_0} - \prod_1^n + \frac{W_{pp}}{W_0} + \frac{W_{pe}}{W_0}\prod_j^n$$

$$\prod_1^n - \frac{W_e}{W_0} = \frac{1}{W_0}\left(W_{pp} + W_{pe}\prod_j^n\right)$$

6. Finally:

$$\boxed{W_0 = \frac{W_{pp} + W_{pe}\prod_j^n}{\prod_1^n - \underbrace{W_e/W_0}_{\text{estimate from correlations}}}}$$

cf (without weight drop)

$$W_0 = \frac{W_{pp}}{1 - W_f/W_0 - \underbrace{W_e/W_0}_{\text{estimate from correlations}}}$$

Or, including fuel safety factor SF:

$$W_0 = \frac{W_{pp} + W_{pe}\left[1 - \mathrm{SF}(1 - \prod_j^n)\right]}{[1 - \mathrm{SF}(1 - \prod_1^n)] \; - \; W_e/W_0}$$

See also Mattingly et al. *Aircraft Engine Design* § 3.2.13.

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Example: Jet ASW aircraft — 3

Revisit jet ASW design with a stores drop requirement added.

In addition to previous mission requirements, carry and drop 400 kg of sonobuoys prior to 3-hour loiter. Keep the 6% 'reserve' fuel requirement.

$M_{pp} = 4900$ kg, and $M_{pe} = 400$ kg. Allow 6% reserve fuel, SF = 1.06.

$$W_0 = \frac{W_{pp} + W_{pe}[1 - SF(1 - \Pi_j^n)]}{[1 - SF(1 - \Pi_1^n)] - W_e/W_0} \quad \text{or} \quad M_0 = \frac{M_{pp} + M_{pe}[1 - SF(1 - \Pi_j^n)]}{[1 - SF(1 - \Pi_1^n)] - M_e/M_0}$$

← correlation $0.88 M_0^{-0.07}$

Stores are dropped at end of segment 3 (cruise); $j = 3$, $n = 7$.

$$\Pi_3^7 = \frac{W_4}{W_3} \times \frac{W_5}{W_4} \times \frac{W_6}{W_5} \times \frac{W_7}{W_6} = 0.928 \times 0.859 \times 0.992 \times 0.995 = 0.787$$

$$\Pi_1^7 = \frac{W_0}{W_1} \times \frac{W_2}{W_1} \times \frac{W_3}{W_2} \times \Pi_3^7 = 0.646 \quad \text{(as found previously with no weight drop).}$$

$$M_0 = \frac{4900 + 400[1 - 1.06(1 - 0.787)]}{[1 - 1.06(1 - 0.646)] - 0.88 M_0^{-0.07}} = \frac{4900 + 400 \times 0.774}{0.625 - 0.88 M_0^{-0.07}} = \frac{5209}{0.625 - 0.88 M_0^{-0.07}} \text{ kg}$$

LHS (kg)	RHS (kg)
26000	26983
26983	26827
26827	26848
26848	26848

→ accept: $\underline{M_0 = 26850}$ kg

cf previous value (no sonobuoys) of 25465 kg : increase of 1385 kg

= 3.46 × 400 kg

A general method

The formal method used for weight drop becomes cumbersome once there are a number of drops (or also, weight gains!).

A related general method (which also works fine when there is no weight drop) is to iterate directly with the weight balance equation. See Nicolai & Carichner § 5.5.

$$W_0 = \underbrace{W_{\text{crew}} + W_{\text{payload}}}_{\text{specified/known}} + \underbrace{W_{\text{fuel}} + W_{\text{empty}}}_{\text{unknown}}$$

1. Start with a guess for W_0.
2. Use a correlation to estimate W_e.
3. Estimate the weight at the end of each segment using historical/Breguet/performance fractions (record difference from the start of the segment, which is fuel weight consumed in each segment).
4. Subtract off dropped weight (or add in acquired weight) at end of any segment as required.
5. At the end of the flight, add up W_e, known/payload/dropped weights and fuel weight for each segment. Apply a safety margin to total fuel use (if that's called for).
6. If the sum is less than guessed W_0, reduce W_0. If it's more, increase W_0.
7. Iterate until equation balances.

This general methodology also works when our estimate for W_e is eventually based on firmer estimates (component weight correlations) rather than a crude initial correlation, and is straightforward to incorporate in spreadsheets or computer programs.

Example: Jet ASW aircraft — 4

ASW jet with stores drop revisited, general method. Drop 400 kg after segment 3.

$M_{pp} = 4900$ kg, $M_{pe} = 400$ kg, $M_e/M_0 = 0.88\, M_0^{-0.07}$ (kg),

$W_3/W_0 = 0.821$, $W_7/W_3 = 0.787$, fuel SF = 1.06.

Guess $\underline{M_0 = 26000 \text{ kg}}$ $\quad M_e = 26000 \times 0.88 \times 26000^{-0.07}$ kg $= 11231$ kg.

$M_3 = 0.821 \times 26000 = 21347$ kg, fuel used 4654 kg.

drop M_{pe} $\quad -400$

$\quad 20946$ kg

$M_7 = 0.787 \times 20946 = 16485$ kg, fuel used 4461 kg

$\quad 9116$ kg

Allow SF = 1.06 : $M_{fuel} = 1.06 \times 9116$ kg $= 9662$ kg

Sum $M_0 = 4900 + 400 + 9662 + 11231 = \underline{26193 \text{ kg}}$ (> guess).

Guess $\underline{M_0 = 27000 \text{ kg}}$ $\quad M_e = 27000 \times 0.88 \times 27000^{-0.07}$ kg $= 11632$ kg.

$M_3 = 0.821 \times 27000 = 22167$ kg, fuel used 4833 kg.

$- M_{pe}$ $\quad -400$

$\quad 21767$ kg

$M_7 = 0.787 \times 21767$ kg $= 17131$ kg, fuel used 4636 kg

$\quad 9469$ kg

Allow SF = 1.06 : $M_{fuel} = 1.06 \times 9469$ kg $= 10038$ kg

Sum $M_0 = 4900 + 400 + 10038 + 11632 = \underline{26970 \text{ kg}}$ (< guess).

Guess $\underline{M_0 = 26850 \text{ kg}}$ $\quad M_e = 26850 \times 0.88 \times 26850^{-0.07} = 11572$ kg.

$M_3 = 0.821 \times 26850 = 22044$ kg, fuel used 4806 kg.

$- M_{pe}$ $\quad -400$

$\quad 21644$ kg

$M_7 = 0.787 \times 21644$ kg $= 17033$ kg, fuel used 4611 kg

$\quad 9417$ kg

Allow SF = 1.06, $M_{fuel} = 1.06 \times 9417$ kg $= 9982$ kg

Sum $M_0 = 4900 + 400 + 9982 + 11572 = \underline{26854 \text{ kg}}$ ✓

Close enough, $\underline{M_0 = 26850 \text{ kg}}$ (same as before).