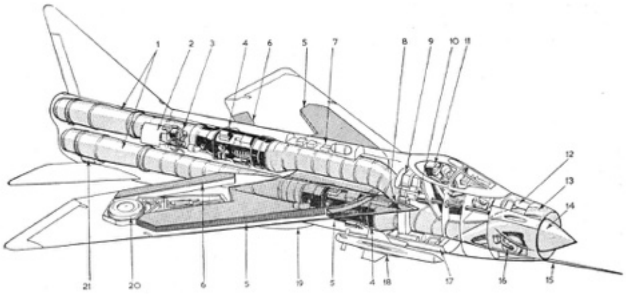
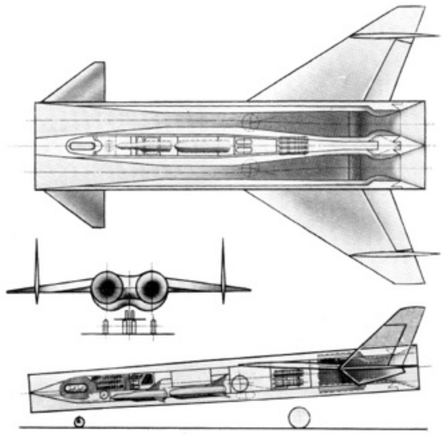
1



Design as inversion of performance analysis



Recommended reading:

Mattingly et al.: Chapter 2
Brandt el al: Chapter 9
Torenbeek: Chapter 5
Jenkinson & Marchman: Chapter 2
Loftin, NASA RP-1060: Chapters 6 and 7

2

Design for performance

1. To carry out design, we have to invert the performance equations to enable the key sizing variables *W, T* and *S* to be estimated from the requirements.
2. For example, to calculate overall weight of fuel required for a mission with a number of segments, we could invert the range equation for specified *R, V, L/D*:

   from $R = \frac{1}{\mathrm{g}c_t} V \frac{L}{D} \ln \frac{W_{i-1}}{W_i}$ invert to obtain $\frac{W_i}{W_{i-1}} = \exp \frac{-R\mathrm{g}c_t}{V(L/D)}$ ***Mission analysis***

   and then find the overall ratio of final to initial weights as $\frac{W_n}{W_0} = \frac{W_1}{W_0} \times \frac{W_2}{W_1} \times \frac{W_3}{W_2} \times \cdots \times \frac{W_n}{W_{n-1}}$
3. Next, most of the performance requirements can be expressed in terms of *W/S* and *T/W* (or *P/W*) at various points in the flight (when the thrust or weight may be different from the takeoff values). We have already used $T = \alpha T_0$ (or $P = \alpha P_0$) for de-rating of thrust/power, where e.g. T_0 is the rated SL static thrust. Now we also introduce weight reduction factor β. i.e. $W = \beta W_0$ where W_0 is the maximum takeoff weight.
4. The basis for most performance-related design turns out to be the Fundamental Performance Equation based on path-tangential energy considerations. Again, we in invert it for use in design:

$$\frac{(T-D)V}{W} = \frac{\mathrm{d}}{\mathrm{d}t}\left(h + \frac{V^2}{2\mathrm{g}}\right) = \frac{\mathrm{d}e}{\mathrm{d}t} = P_s$$

(weight-specific) excess power *Ps*.

$$\frac{T}{W} = \frac{D}{W} + \frac{P_s}{V}$$

$$\frac{\alpha T_0}{\beta W_0} = \frac{D}{\beta W_0} + \frac{P_s}{V}$$

$$\frac{T_0}{W_0} = \frac{\beta}{\alpha}\left(\frac{D}{\beta W_0} + \frac{P_s}{V}\right)$$

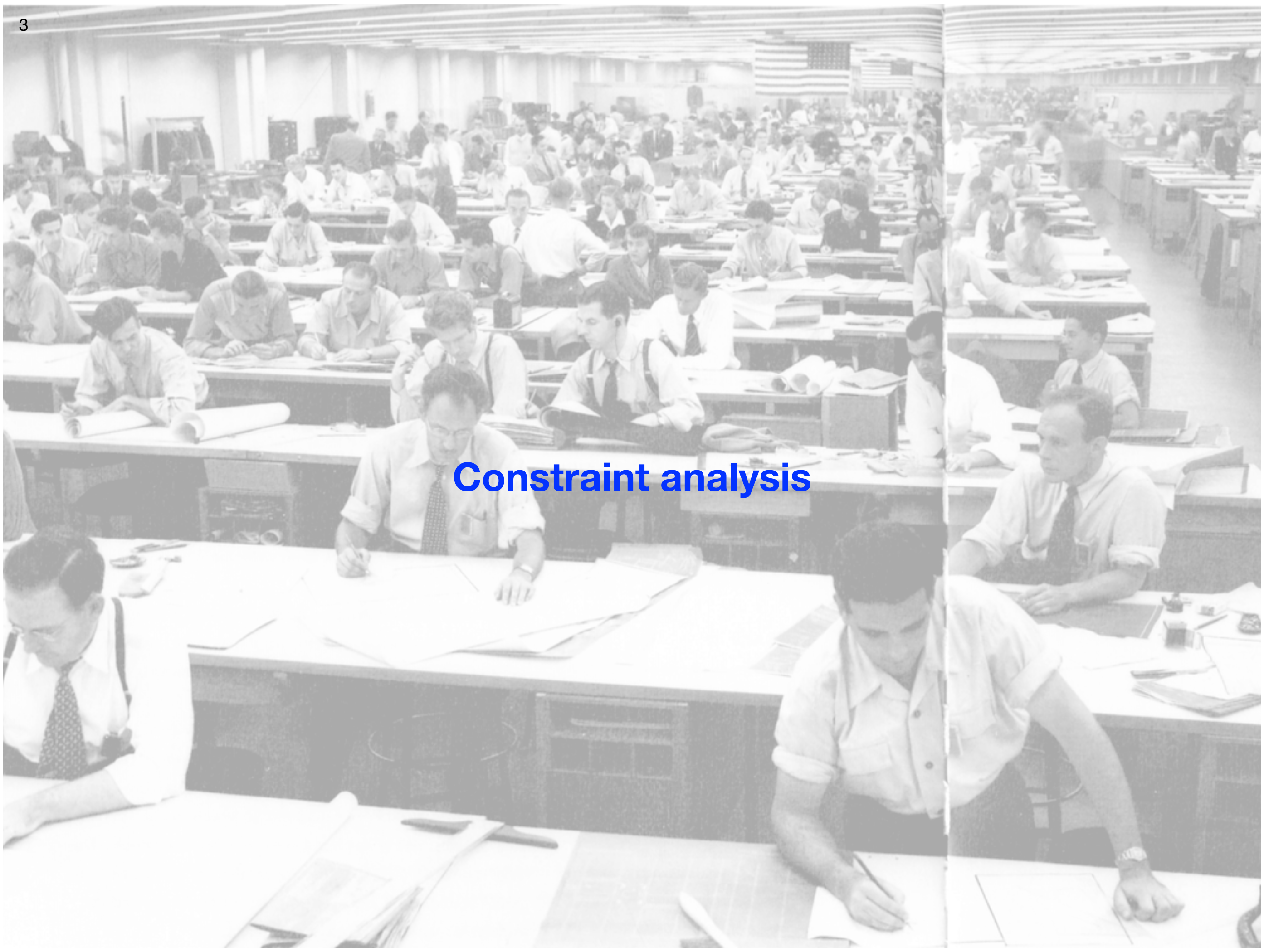
Now $C_D = C_{D,0} + KC_L^2$ and $C_L = \frac{n\beta W_0}{qS}$

so $C_D = C_{D,0} + K\left(\frac{n\beta}{q}\frac{W_0}{S}\right)^2$

Finally $$\frac{T_0}{W_0} = \frac{\beta}{\alpha}\left(\frac{qS}{\beta W_0}\left[C_{D,0} + K\left(\frac{n\beta}{q}\frac{W_0}{S}\right)^2\right] + \frac{P_s}{V}\right)$$

expresses a design constraint between (takeoff) thrust loading and wing loading.

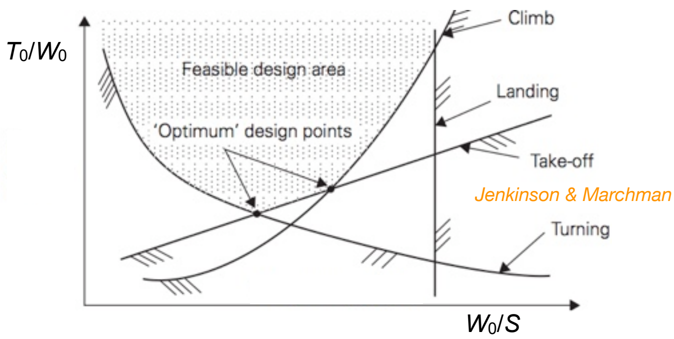
Constraint analysis



Constraint analysis

Constraints based on performance requirements

1. At this point we have established an initial weight estimate W_0 based on fuel use and expendable payload, and, if required, we used preliminary/historical values of β and W_0 together with some of the (non-Breguet-based) performance requirements that might require using a significant weight of fuel.
2. We now turn attention to finding sensible choices for T_0/W_0 (or P_0/W_0 in the case of piston/prop) and W_0/S, based on the performance requirements (those which just involve these ratios). <u>This stage is called constraint analysis</u>.
3. Constraint analysis requires us to plot all significant requirements that directly involve wing and/or thrust loading on a single graph. We always need to use α and β to relate the performance requirements back to take-off values (since that's what we want) in the standard atmosphere.
4. Note that the empty weight fraction correlations for W_e/W_0 used up to this point did not explicitly involve T_0/W_0 or W_0/S, so that weight estimation was independent of performance analysis. More generally, the empty weight fraction *does* involve these parameters, and so the two sets of analyses would be no longer independent. The initial analysis we do here is "first-order", without coupling.
5. Just as the performance requirements could intrude into the weight estimation stage, so the mission-phase weight estimates (in the form of fractions $\beta_i = W_i/W_0$) typically intrude into the constraint analysis. Hence, again, as we refine the design we may need to iterate.
6. We generally wish to have small values of T_0/W_0 and large values of W_0/S to minimize cost (size).



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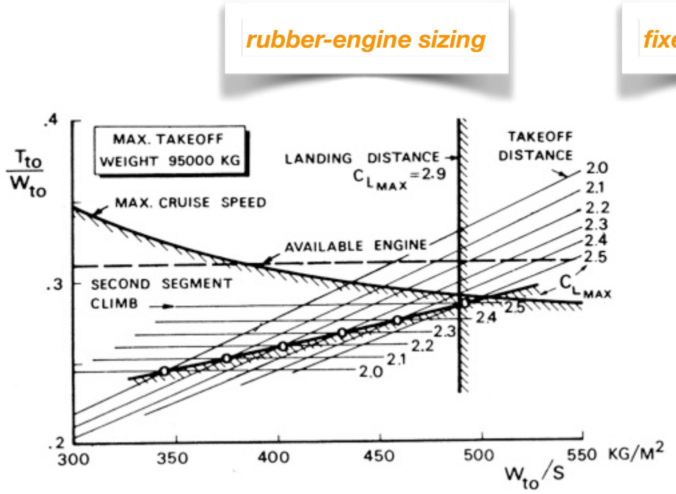
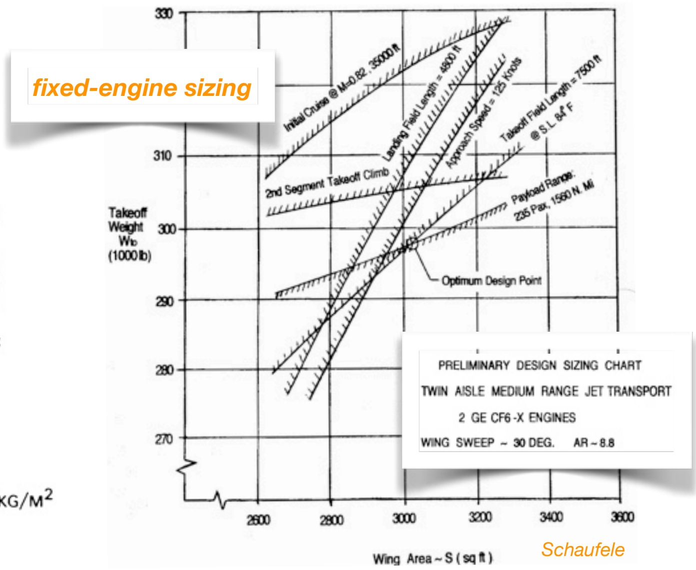
“Rubber-engine” vs “fixed-engine” sizing

1. Unlike wing areas and aircraft weights, powerplants come in a restricted number of sizes.
2. In recognition of this fact, there are two ways to proceed with constraint analysis:
   a. “Rubber-engine” sizing, where we make our plot in terms of the ratios W_0/S and T_0/W_0 and either assume we can buy off the shelf or stretch an existing engine to give a required value of T_0, and;
   b. “Fixed-engine” sizing, where we take existing engines (perhaps a few alternatives) and their values of T_0, and for each engine alternative produce a constraint plot with W_0 and S as the axes.
3. Either of these options is fine, and has its strengths. In the first iteration at least it is probably best to stay with “rubber-engine” sizing and show available engines as lines of constant T_0/W_0 on the plot.



Torenbeek



Schaufele

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Constraint examples — 1

Most (but not all) performance analysis can be stated in terms of the master constraint equation, which is simply a re-arrangement of the fundamental performance equation.

$$\frac{T_0}{W_0} = \frac{\beta}{\alpha}\left(\frac{qS}{\beta W_0}\left[C_{D,0} + K\left(\frac{n\beta}{q}\frac{W_0}{S}\right)^2\right] + \frac{P_s}{V}\right) \qquad q = \tfrac{1}{2}\rho V^2 = \tfrac{1}{2}\gamma p M^2 \qquad P_s = \frac{\mathrm{d}}{\mathrm{d}t}\left(h + \frac{V^2}{2\mathrm{g}}\right)$$

Since we want a relationship between T_0/W_0 and W_0/S, we must have all the other values given, or be able to derive them.

We note that $C_L = \dfrac{n\beta W_0}{qS}$ and $C_{D,0} + K\left(\dfrac{n\beta}{q}\dfrac{W_0}{S}\right)^2 \equiv C_D$

If appropriate (e.g. when external stores are carried) we should increase $C_{D,0}$. Similarly, if the aircraft is in take-off or landing configuration, $C_{D,0}$ should be increased to take account of landing gear and flap deployment. The coefficient K should also be re-assessed for different flight conditions.

The examples below are generally similar but in places somewhat simpler than those presented in Mattingly et al. Ch. 2. They are based around a thrust-related propulsion model.

1. Constant Altitude/Speed Cruise ($P_s = 0$) Given dh/dt = 0, dV/dt = 0, n = 1 (L = W), values of h and V (i.e. q) and with α, β.

$$\frac{T}{W} = \frac{\alpha T_0}{\beta W_0} = \frac{D}{L} \qquad \text{or} \qquad \frac{T_0}{W_0} = \frac{\beta}{\alpha}\left(C_{D,0}\frac{q}{\beta(W_0/S)} + K\frac{\beta W_0/S}{q}\right) = \frac{\beta}{\alpha}\left(\frac{C_{D,0}}{C_L} + KC_L\right)$$

We find that T_0/W_0 becomes very large either for small or large values of W_0/S, with a minimum at

$$\left[\frac{W_0}{S}\right]_{\min T/W} = \frac{q}{\beta}\sqrt{\frac{C_{D,0}}{K}} \qquad \text{and} \qquad \left[\frac{T_0}{W_0}\right]_{\min} = \frac{\beta}{\alpha}2\sqrt{C_{D,0}K} \qquad \text{i.e. at } (L/D)^*.$$

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Constraint examples — 2

Note that for the cruise constraint it is advisable to plot C_L as a parameter along the curve (so we can assess if we are near to one of the optimum values we desire, e.g. that will give $(C_L^{1/2}/C_D)_{max}$.

2. Constant Speed Climb Given dV/dt = 0, n≈1 ($L \approx W$), values of h, dh/dt > 0 and V (i.e. q) and with α, β:

$$\frac{T_0}{W_0} = \frac{\beta}{\alpha}\left(C_{D,0}\frac{q}{\beta(W_0/S)} + K\frac{\beta W_0/S}{q} + \frac{1}{V}\frac{\mathrm{d}h}{\mathrm{d}t}\right) = \frac{\beta}{\alpha}\left(\frac{C_{D,0}}{C_L} + KC_L + \frac{1}{V}\frac{\mathrm{d}h}{\mathrm{d}t}\right)$$

Like case 1, we find that T_0/W_0 becomes very large either for small or large values of W_0/S, with a minimum at

$$\left[\frac{W_0}{S}\right]_{\min T/W} = \frac{q}{\beta}\sqrt{\frac{C_{D,0}}{K}} \quad \text{and} \quad \left[\frac{T_0}{W_0}\right]_{\min} = \frac{\beta}{\alpha}\left(2\sqrt{C_{D,0}K} + \frac{1}{V}\frac{\mathrm{d}h}{\mathrm{d}t}\right)$$

3. Constant Speed, Constant Altitude Turn (Ps=0) Given dV/dt = 0, dh/dt = 0, values of n > 1, h, and V (i.e. q) and with α, β.

$$\frac{T_0}{W_0} = \frac{\beta}{\alpha}\left(C_{D,0}\frac{q}{\beta(W_0/S)} + K\frac{n^2\beta W_0/S}{q}\right)$$

Another curve with a turning point, now at

$$\left[\frac{W_0}{S}\right]_{\min T/W} = \frac{q}{n\beta}\sqrt{\frac{C_{D,0}}{K}} \quad \text{and} \quad \left[\frac{T_0}{W_0}\right]_{\min} = \frac{n\beta}{\alpha}\left(2\sqrt{C_{D,0}K}\right)$$

If instead given either rate ω or radius of turn R_H instead, compute n from

$$n = \sqrt{1 + \left(\frac{\omega V}{\mathrm{g}}\right)^2} \quad \text{or} \quad n = \sqrt{1 + \left(\frac{V^2}{\mathrm{g}R_H}\right)^2}$$

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Constraint examples — 3

4. Horizontal Acceleration (P_s = (V/g)(dV/dt)) Given dV/dt > 0, n=1 (L=W), values of h, dh/dt = 0 and V (i.e. q) and with α, β. Similar to case 2:

$$\frac{T_0}{W_0} = \frac{\beta}{\alpha}\left(C_{D,0}\frac{q}{\beta(W_0/S)} + K\frac{\beta W_0/S}{q} + \frac{1}{\mathrm{g}}\frac{\mathrm{d}V}{\mathrm{d}t}\right)$$

If instead of dV/dt, we are given an initial and final speed and a minimum time allowed for the change in speed, the analysis becomes more complex, requiring iterative solution. See Mattingly et al.

5. Take-Off Ground Roll s_G Given dh/dt = 0, $C_{L,max}$, $V_{TO} = k_{TO}V_{stall}$, μ_{TO}. Here we have to change the notation slightly to allow for tyre drag and the fact that this will vary with lift (see our earlier analysis of ground roll). Note that also as noted previously, $C_{D,0}$ should be increased owing to landing gear and flap deployment.

Given a value of s_G, we have to solve the following equation for each W_0/S:

$$s_G = -\frac{\beta(W_0/S)}{\rho\,\mathrm{g}\,\xi_{TO}}\ln\left\{1 - \xi_{TO}/\left[\left(\frac{\alpha}{\beta}\frac{T_0}{W_0} - \mu_{TO}\right)\frac{C_{L\max}}{k_{TO}^2}\right]\right\} \quad \text{where} \quad \xi_{TO} = C_{D,0} - \mu_{TO}\,C_{L\max}$$

See Mattingly et al. for case where in addition an obstacle of specified height must be cleared.

6. Landing Ground Roll s_G Given dh/dt = 0, $C_{L,max}$, $V_{TD} = k_{TD}V_{stall}$, and μ_B. Similar to the previous case but, if there is no allowance for thrust reversal, simpler. Note that $C_{D,0}$ and $C_{L,max}$ should be assessed in landing configuration.

$$s_G = -\frac{\beta(W_0/S)}{\rho\,\mathrm{g}\,\xi_L}\ln\left\{1 - \xi_L/\left[\mu_B\frac{C_{L\max}}{k_{TD}^2}\right]\right\} \quad \text{where} \quad \xi_L = C_{D,0} - \mu_B\,C_{L\max}$$

See Mattingly et al. for cases with thrust reversal and obstacle of specified height.

Constraint examples — 4

7. Service Ceiling ($P_s = dh/dt$) Given $dV/dt = 0$, $n=1$ ($L = W$), values of h, $dh/dt > 0$ and C_L, with α, β. Note that this analysis assumes that C_L is given (say cruise value) whereas in general it could be varied in order to maximise rate of climb or reduce the required thrust. In that case the analysis would be more complicated.

First $V = \sqrt{\frac{2\beta}{\rho C_L}\frac{W_0}{S}}$ then $\frac{T_0}{W_0} = \frac{\beta}{\alpha}\left(\frac{C_{D,0}}{C_L} + KC_L + \frac{1}{V}\frac{\mathrm{d}h}{\mathrm{d}t}\right)$

8. Take-off Climb Angle Given θ, $n = 1$, $dV/dt = 0$ and $C_{L,\max}$ (in take-off configuration), k_{TO}, with α, β.

$$\frac{T_0}{W_0} = \frac{\beta}{\alpha}\left(\frac{k_{TO}^2 C_{D,0}}{C_{L\max}} + K\frac{C_{L\max}}{k_{TO}^2} + \sin\theta\right)$$

where $V_{TO} = \sqrt{\frac{2\beta}{\rho}\frac{W_0}{S}\frac{k_{TO}^2}{C_{L\max}}}$ is used to find α.

9. Approach Speed Given V_{app}, $dh/dt \approx 0$ and $C_{L,\max}$ (in landing configuration), k_{app}, with β.

$$\left[\frac{W_0}{S}\right]_{\max} = \frac{\rho\, C_{L\max} V_{\mathrm{app}}^2}{2\,\beta\, k_{\mathrm{app}}^2}$$

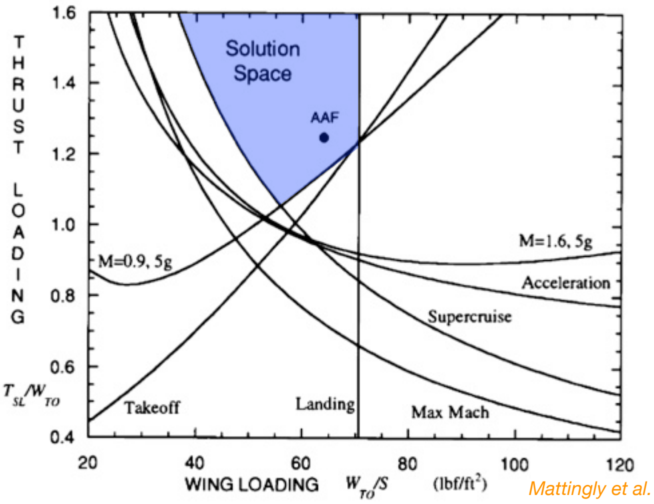
Note that thrust is assumed small. This constraint gives a vertical line on the plot.

The above is by no means a complete set of possible performance constraints but it is sufficient as a starting point for further examination. In general one must examine carefully all the requirements to consider how they can either be framed as performance constraints or how they contribute to mission analysis.

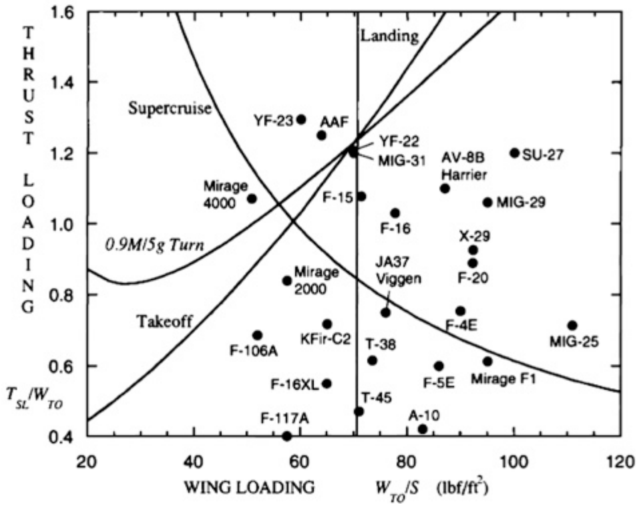
If we have a propeller-powered aircraft, then the appropriate form of the master constraint equation is

$$\frac{P_0}{W_0} = \frac{\beta}{\eta_{\mathrm{pr}}\alpha}\left\{V\left[\frac{q}{\beta(W_0/S)}C_{D,0} + n^2\frac{\beta(W_0/S)}{q}K\right] + P_s\right\}$$

Constraint examples — 5



Mattingly et al.



Finally, after all the constraints have been obtained, they are plotted on a single diagram of T_0/W_0 vs W_0/S. Taking account of the fact that the constraints generally amount to inequalities rather than equations, there is a space of feasible solutions (the solution space) Generally we pick low feasible values of T_0/W_0 and high feasible values of W_0/S in order to minimise size and cost. At this point, recognizing that our values of β are rather imprecise, it is best to be slightly conservative.

It is good practice to also plot the values of T_0/W_0 vs W_0/S obtained for comparable designs. While their design constraints may have been different, the comparison is revealing and we may be able to guess how these differed from what we have used. Also this is a useful 'sanity check'. Finally, we can at this stage select the determining constraints for further analysis, allowing us to concentrate on the most important cases.

We now have estimates for W_0, T_0 and S.

Targetting best Range/Endurance — 1

$$\frac{W_0}{S} = \frac{q}{\beta} C_L$$

1. So far our performance constraint analysis has been uncoupled from range/endurance requirements, but these too can imply a target wing loading if a cruise/loiter (height, speed) pair are given (i.e. q). The target wing loading will be associated with an optimal point on the drag polar and hence will depend on aerodynamic parameters $C_{D,0}$ and K.

Maximise	Jet	Prop
Range	$(C_L^{1/2}/C_D)_{max}$	$(C_L/C_D)_{max}$
Endurance	$(C_L/C_D)_{max}$	$(C_L^{3/2}/C_D)_{max}$

Maximise	Jet	Prop
Range	$\frac{W_0}{S} = \frac{q}{\beta}\sqrt{\frac{C_{D,0}}{3K}}$	$\frac{W_0}{S} = \frac{q}{\beta}\sqrt{\frac{C_{D,0}}{K}}$
Endurance	$\frac{W_0}{S} = \frac{q}{\beta}\sqrt{\frac{C_{D,0}}{K}}$	$\frac{W_0}{S} = \frac{q}{\beta}\sqrt{\frac{3C_{D,0}}{K}}$

2. The amount of thrust/power required to fly steady and level at the optimal wing loading can also be determined, producing a single point on the constraint line for cruise (a.k.a. "payload-range") constraint.

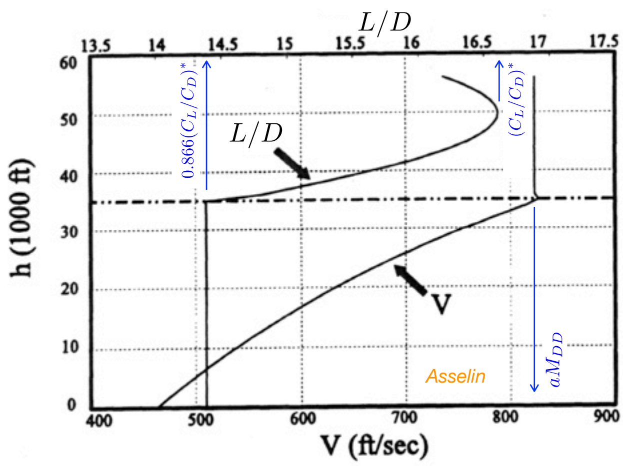
Maximise	Jet	Prop
Range	$\frac{T_0}{W_0} = \frac{\beta}{\alpha}\left(\frac{16}{3}C_{D,0}K\right)^{1/2}$	$\frac{P_0}{W_0} = \frac{\beta}{\eta_{pr}\alpha}\left(4\frac{2}{\rho}\frac{\beta W_0}{S}\right)^{1/2} C_{D,0}^{1/4}K^{3/4}$
Endurance	$\frac{T_0}{W_0} = \frac{\beta}{\alpha}(4C_{D,0}K)^{1/2}$	$\frac{P_0}{W_0} = \frac{\beta}{\eta_{pr}\alpha}\left(\frac{16}{3}\frac{2}{\rho}\frac{\beta W_0}{S}\right)^{1/2} C_{D,0}^{1/4}K^{3/4}$

3. A reasonable way to proceed is to note the value of C_L at the required optimum point on the drag polar and then to plot C_L as a parameter along the cruise constraint curve. If we are fortunate, then this optimal C_L will be in the feasible region on the constraint plot and we could aim for the corresponding location as our (T_0/W_0, W_0/S) pair. If not, we may be free to choose a different altitude (the 'best cruise altitude' or BCA) at which to cruise (which will influence C_L along the curve). If neither is true then we will have to re-compute the fuel usage based on achievable values of C_L, rather than the assumed optimal values.

Targetting best Range/Endurance — 2

4. We note that the above tabulated values of wing and thrust loading (in particular, the location on the drag polar for which they are computed) are made without regard to compressibility effects. For most applications except perhaps the most practically significant, i.e. range of transonic jet aircraft, this is typically not important.

5. However, in the case of subsonic jet aircraft, there is a rapid rise in C_D near M_{DD}, if enough thrust is available to approach this speed (as is often the case). This effectively limits efficient cruise speed to M_{DD}. With M limited to this value, ML/D can only be increased by increasing C_L towards C_L* and L/D towards (L/D)*.



6. A key jet aircraft design objective is to arrange the aerodynamics and wing loading such that cruise at M_{DD} corresponds to the point on the drag polar that maximises ML/D. This may require changing the cruise altitude but care must be taken to also minimise fuel consumption (c_t).

Constant altitude, constant speed cruise – revisited

Effects of altitude, speed, drag polar parameters.

JET

$$\frac{T_0}{W_0} = \frac{\beta}{\alpha}\frac{1}{C_L/C_D} = \frac{\beta}{\alpha}\left(\frac{C_{D,0}}{C_L} + \frac{C_L}{\pi A e}\right)$$

$$C_L = \frac{\beta}{\frac{1}{2}\rho V^2}\frac{W_0}{S} = \frac{\beta}{q}\frac{W_0}{S}$$

Turning point at:

$$\left(\frac{W_0}{S}\right)_{(T/W)_{\min}} = \frac{q}{\beta}C_L^* = \frac{\frac{1}{2}\rho V^2}{\beta}\sqrt{C_{D,0}\pi A e}$$

T.P. moves to higher W/S for larger *q, A,* $C_{D,0}$.

$$\left(\frac{T_0}{W_0}\right)_{\min} = \frac{\beta}{\alpha}\frac{1}{(C_L/C_D)^*} = 2\frac{\beta}{\alpha}\sqrt{\frac{C_{D,0}}{\pi A e}}$$

T.P. moves to higher T/W for larger $C_{D,0}$ and smaller *A*, *no altitude or speed effect.*

Generally:

$$\frac{T_0}{W_0} = \frac{\beta}{\alpha}\left[\frac{q}{\beta(W_0/S)}C_{D,0} + \frac{\beta(W_0/S)}{q}\frac{1}{\pi A e}\right]$$

PROP

$$\frac{P_0}{W_0} = \frac{\beta}{\eta_{\text{pr}}\alpha}\frac{1}{C_L/C_D}V = \frac{\beta}{\eta_{\text{pr}}\alpha}\frac{C_D}{C_L}\sqrt{\frac{2}{\rho}\beta\frac{W_0}{S}\frac{1}{C_L}} = \frac{\beta}{\eta_{\text{pr}}\alpha}\left(\frac{C_{D,0}}{C_L} + \frac{C_L}{\pi A e}\right)\sqrt{\frac{2}{\rho}\beta\frac{W_0}{S}\frac{1}{C_L}}$$

For minimum power, $C_L = \sqrt{3}C_L^* = \sqrt{3C_{D,0}\pi A e}$

Turning point at:

$$\left(\frac{W_0}{S}\right)_{(P/W)_{\min}} = \frac{q}{\beta}\sqrt{3C_{D,0}\pi A e}$$

T.P. moves to higher W/S for larger *q, A,* $C_{D,0}$.

$$\left(\frac{P_0}{W_0}\right)_{\min} = \frac{\beta}{\eta_{\text{pr}}\alpha}\left(\frac{256}{27}\frac{C_{D,0}}{(\pi A e)^3}\right)^{1/4}\left(\frac{2}{\rho}q\sqrt{3C_{D,0}\pi A e}\right)$$

T.P. moves to higher P/W for larger $C_{D,0}$ and smaller *A*, *both altitude and speed effect.*