

Introduction to aircraft stability

Torenbeek & Wittenberg Ch 7 Anderson Ch 7



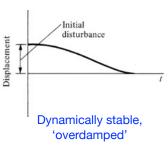
Stability - static and dynamic

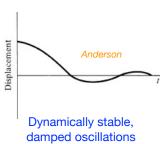
Statically stable Statically neutrally stable

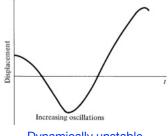
Definitions

If the forces/moments on a body resulting from a fixed (static) disturbance to an equilibrium are such as to tend to bring about a return to equilibrium, the body has static stability.

A body is dynamically stable if the force/moments acting on it produce over time a return to and maintenance of an equilibrium position. This can involve both positiondependent and velocity-dependent forces.







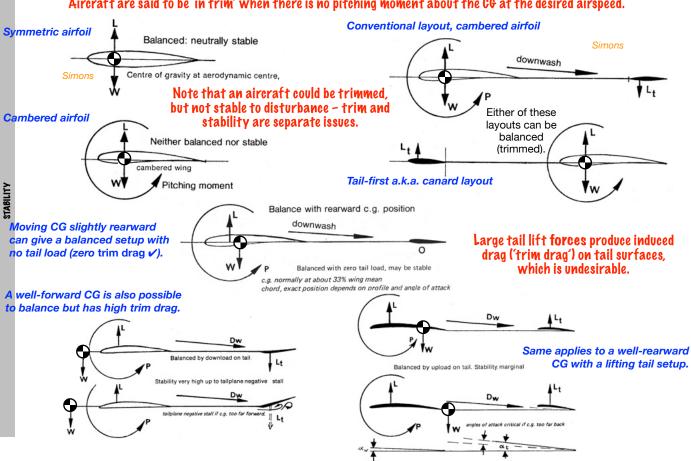
Dynamically unstable

Static stability is necessary for dynamic stability to be possible. However static stability is not sufficient to ensure dynamic stability. (Flutter is one example: the aircraft structure by its stiffness typically provides forces that tend to restore static equilibrium, but aerodynamics may provide velocity-dependent forces that tend to excite rather than suppress motion - these are not present for a static displacement.)

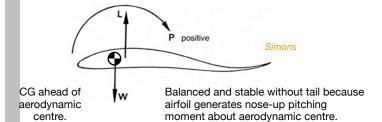
Even more fundamentally we are typically primarily concerned with the static longitudinal (pitch) stability of aircraft. This relates to the maintenance of an aircraft at a given angle of attack (or, equivalently, C_L).

Aircraft layout and trim or longitudinal balance

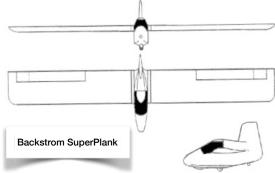
Aircraft are said to be 'in trim' when there is no pitching moment about the CG at the desired airspeed.

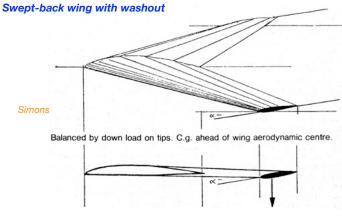


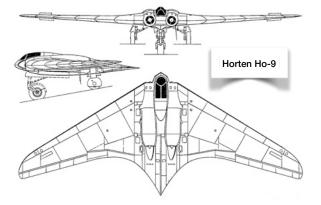




Reflexed or negatively cambered airfoil



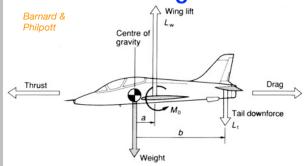




Can be balanced because wing-tips generate downforce rear of CG.

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Longitudinal trim and (static) stability

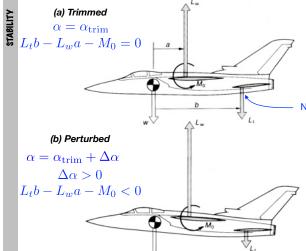


An aircraft is said to be trimmed if all the moments about the CG sum to zero – there is no tendency for the aircraft to rotate about the centre of mass.

For the situation pictured (and recalling the convention that a positive moment gives a nose-up pitch, while a positively cambered airfoil will give a nose-down pitching moment about its aerodynamic center):

$$L_t b - L_w a - M_0 = 0$$

Note that here we have ignored pitching moments that thrust and drag might in reality contribute, by assuming they pass through the CG.



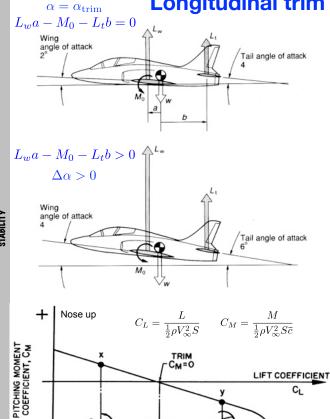
The aircraft above is trimmed in pitch, but perhaps not necessarily statically stable: that would also require that a disturbance (say a nose-up perturbation) generates a restoring moment.

Note that tail is set at a lower angle of attack than the wing and so has negative lift.

The diagrams to the left shows that the set-up was in fact statically stable, as well. For an aircraft with the same arrangement that is trimmed in situation (a), a nose-up disturbance as seen in (b) will increase wing lift L_w and decrease tail downforce L_t while leaving M_0 unaltered. The net effect is to provide a nose-down moment around the CG, tending to restore the equilibrium.

Longitudinal trim and (static) stability

 C_L



Hurt

Nose down

We also see that set-ups are possible the provide trimmed flight, but which are statically unstable to disturbance: a nose-up perturbation generates a moment that tends to pitch the nose up still further. Typically in these unstable cases the CG is more rearward, and the tail force is upwards (providing a nose-down pitching moment when trimmed).

By contrast, in the statically stable case, the tail force is typically downwards (providing a nose-up pitching moment when the aircraft is trimmed).

Recognizing that in steady level flight the aircraft has to generate a positive lift, $C_L > 0$, we can summarize the situation for static stability and trim on a C_M vs C_L (or pitching moment) diagram.

Here, C_M is the coefficient of all the moments around the aircraft CG position.

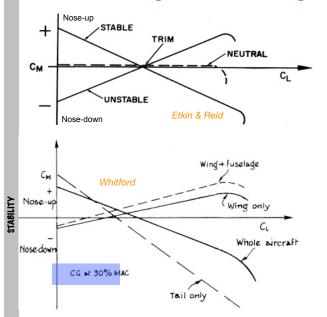
Since for $C_L < C_{L\text{max}}$, $C_L \propto a$, this could equally well be drawn as a C_M vs α diagram (and often is).

Either way: there is one particular positive C_L for which the aircraft is trimmed in pitch, and the slope of the line relating C_M to C_L has to be negative.

Summary: for a trimmed, statically stable aircraft,

$$C_{M,C_L=0}>0$$
 and $rac{\mathrm{d}C_M}{\mathrm{d}C_L}<0$

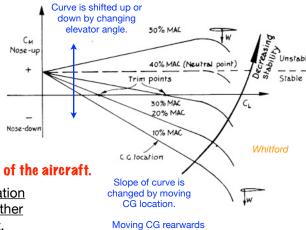
Pitching moment diagram and static stability margin



Aircraft set-ups with negative and neutral stability can equally well be represented on the C_M vs C_L (or *pitching moment*) diagram.

Various aircraft components make their individual contributions to the overall C_M vs C_L curve. Typically all components are destabilizing except the horizontal tail (also called the horizontal stabilizer).

Note that curves on the pitching moment diagram are drawn with respect to one particular location of the CG with respect to the mean aerodynamic chord (MAC) for the aircraft.



makes slope more positive (and decreases stability).

In fact the location of the CG is a key determinant of both the degree of stability (essentially the *slope* of the curve) and the aircraft's trimmed C_L .

The (negative) slope of this line is called the STATIC MARGIN of the aircraft.

Typically the elevator angle setting controls the vertical location of the curve, while the CG location controls its slope. Together these two aspects are used to stabilize and trim the aircraft.

Establishing trim and stability in glide testing

Trimmed C_L must fall in operating range $0 < C_{L,trim} < C_{L,max}$.

If aircraft tries to stall repeatedly then the trimmed C_L exceeds C_{Lmax} . Solutions: (1) add some down elevator or (2) move CG forward.

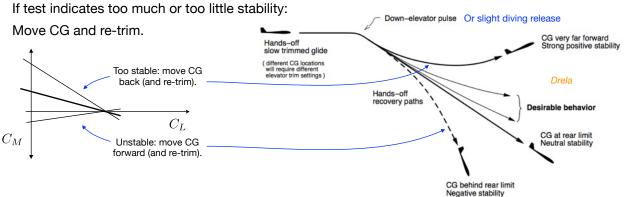


 $C_{L_{
m max}}$ Up elevator $C_{L_{
m max}}$ C_{L} Down elevator $C_{L_{
m max}}$ C_{L}

CG forward

Aircraft must be stable in pitch as well as trimmable.

The 'dive test', conducted once plane has been trimmed to equilibrium.



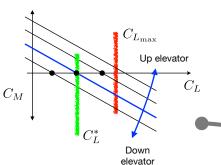
STABILITY

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Moving around on the drag polar

To put it another way: changing trim in level flight changes the flying speed and also shifts location on the aircraft drag polar.

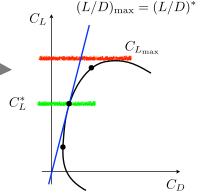
One key motivation for doing this is to produce different trimmed flight speeds.



Once we can trim the plane to fly stably, we can vary C_L and hence get to any/desired points on the drag polar curve, e.g. to $(L/D)^*$.

In turn, this gives us different flying speeds, since $V = \sqrt{\frac{2}{\rho} \frac{W}{S} \frac{1}{C_L}}$

The primary means for changing trim this is to use different elevator settings.



However, it is also possible to vary trimmed C_L by moving the CG location (and hence the negative slope of the C_M/C_L line) – which is the approach used by hang gliders.



A demerit of moving the CG (as opposed to changing elevator deflection) to change the trimmed air speed is that it varies the stability characteristics with trimmed C_L .

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Neutral point and static margin

The fact that the pitching moment diagram depends on the CG location is a little unsatisfactory. However, there is one comparatively simple way of summing up the aircraft's stability in pitch.

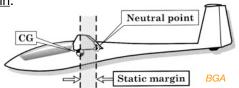
(Recall) for an airfoil or wing there is a particular point or axis of rotation where the aerodynamic pitching moment coefficient is independent of angle of attack (the aerodynamic centre). There is an equivalent axis for the whole aircraft. All the lift can be taken to act at this point.

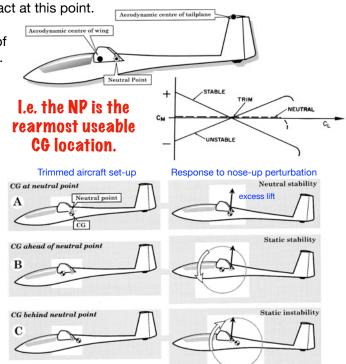
This location, the **neutral point** (NP), is independent of CG position – it is determined by aerodynamics alone.

If the CG is also located at this point, the aircraft is neutrally stable in pitch, so cannot be trimmed to remain at a particular value of C_L or angle of attack.

We adjust the stability (slope of the C_M/C_L curve) by moving the CG around (e.g. by adding mass at the front or rear). And obtain the desired trimmed C_L (or speed) by setting elevator angle.

Stable settings *always* have the CG in front of the neutral point of the aircraft. The distance between the CG and the neutral point (in units of aerodynamic mean chord) is the <u>static</u> (stability) margin.





Aerodynamic centre of a lifting surface

As for an airfoil, the *aerodynamic centre* (or a.c.) of a wing is the moment reference point for which the pitching moment coefficient C_M is independent of the angle of attack.

If the airfoil section is the same along the span and the wing is untwisted then the *mean aerodynamic chord* (MAC) \overline{c} and its spanwise location \overline{y} are given by the integral expressions

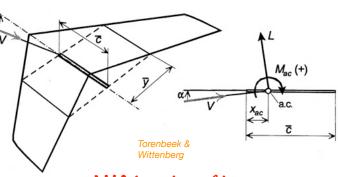
$$\bar{c} = \frac{2}{S} \int_0^{b/2} c^2 \, dy$$
 $\bar{y} = \frac{2}{S} \int_0^{b/2} c \, y \, dy$

For a simple trapezoidal wing shape, these integrals provide

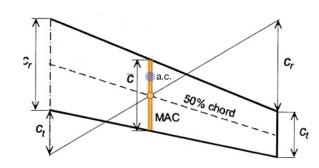
$$\frac{\bar{c}}{c_r} = \frac{2(1+\lambda+\lambda^2)}{3(1+\lambda)} \qquad \quad \frac{\bar{y}}{b/2} = \frac{1+2\lambda}{3(1+\lambda)}$$

Those values as well as the longitudinal location of the MAC can be found for a trapezoidal wing shape using a simple geometric construction:

The longitudinal location of the aerodynamic centre on the wing, x_{ac} , is typically close to $\bar{c}/4$.



MAC depends on c² because airfoil moments per unit span also depend on c².



Tail volume coefficients

where the taper ratio $\lambda = \frac{c_t}{c_r}$

When it comes to their stabilising effect, both the area and moment arm (from either the CG or the a.c. of the wing MAC) of tail surfaces are important – we could trade area for moment arm and get similar effect.

The product of area and moment arm is a volume and the *tail volume coefficients* of the horizontal and vertical tails are dimensionless values of these volumes:

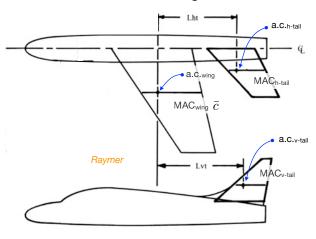
$$V_{
m ht} = rac{L_{
m ht}S_{
m ht}}{ar cS}$$

$$V_{
m vt} = rac{L_{
m vt}S_{
m vt}}{bS}$$

Note that the length scale used in the denominator is different in the two cases.

While there are design methods to establish the values $V_{\rm ht}$ and $V_{\rm vt}$ required for a particular aircraft, one finds that within a class of aircraft the values are rather similar. Here is a table of representative/indicative values.

The fact that in general V_{vt} values are significantly smaller than V_{ht} values reflects the different length scales in the denominators of the definitions (although also in general $S_{vt} < S_{ht}$).



Typical values

	Horizontal $V_{ m ht}$	Vertical $V_{ m v}$
Sailplane	0.50	0.02
Homebuilt	0.50	0.04
General aviation—single engine	0.70	0.04
General aviation—twin engine	0.80	0.07
Agricultural	0.50	0.04
Twin turboprop	0.90	0.08
Flying boat Raymer	0.70	0.06
Jet trainer	0.70	0.06
Jet fighter	0.40	0.07
Military cargo/bomber	1.00	0.08
Jet transport	1.00	0.09

Consider the (nose-up) pitching moment about the CG contributed by the wing alone

$$M_w=C_{M_{ac_w}}\tfrac12\rho V^2S\bar c+C_{L_w}\tfrac12\rho V^2S(h-h_{nw})\bar c$$
 or, dimensionlessly,
$$C_{M_w}=C_{M_{ac_w}}+C_{Lw}(h-h_{nw})$$

The (dimensionless) contribution of the horizontal tail (neglecting its aerodynamic pitching moment w.r.t. its own aerodynamic centre – which is small compared to the lift-generated contributions):

$$-\frac{\bar{l}_{ht}S_{ht}}{\bar{c}S}C_{Lt} + C_{Lt}\frac{S_{ht}}{S}(h - h_{nw}) = -V_{ht}C_{Lt} + C_{Lt}\frac{S_{ht}}{S}(h - h_{nw}) \qquad \qquad \text{recall} \ \ V_{ht} = \frac{\bar{l}_{ht}S_{ht}}{\bar{c}S}$$

Summing contributions of wing and tail to moments about CG

$$C_{M,CG} = C_{M_{ac_w}} + C_{Lw}(h - h_{nw}) - V_{ht}C_{Lt} + C_{Lt}\frac{S_{ht}}{S}(h - h_{nw})$$

$$C_{M,CG} = C_{M_{ac_w}} + C_L(h - h_{nw}) - V_{ht}C_{Lt} \qquad \text{where} \quad C_L = C_{Lw} + C_{Lt}\frac{S_{ht}}{S}$$

or

Locating the neutral point

zero by definition

Differentiating w.r.t. C_L : $\frac{\partial C_{M,CG}}{\partial C_L} = \underbrace{\frac{\partial C_{M,CG}}{\partial C_L}} + (h - h_{nw}) - V_{ht} \frac{\partial C_{Lt}}{\partial C_L}$

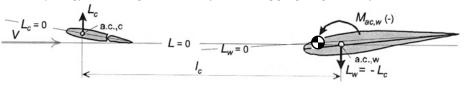
For stability at trimmed condition we know that this slope must be negative, $(h-h_{nw})-V_{ht}\frac{\partial C_{Lt}}{\partial C_L}<0$

and at the limit where there is neutral stability and the CG is at the NP: $(h_{np}-h_{nw})=V_{ht}\frac{\partial C_{Lt}}{\partial C_L}$

We recall that for trimmability we also require a positive pitching moment at zero overall lift, and for a conventional layout this means that the horizontal tail will lift downwards while the wing lifts upward at zero overall lift. This in turn means that the tail is typically set at a negative rigging angle relative to the wing so that there is an overall nose-up moment about the CG at zero lift.



For a canard layout this situation is reversed and at zero overall lift the wing provides a downward lifting force and the (lifting) fore-wing is set at a positive rigging angle compared to the main wing.



We can estimate the lift curve slopes of the isolated wing and tail surfaces, (reduced from airfoil values owing to the downwash generated by tip vortices) e.g. (recall) by

$$\frac{\partial C_L}{\partial \alpha} = \frac{2\pi}{1+2/A} \qquad \quad \text{for unswept wings}$$

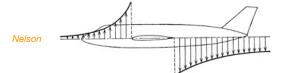
We also have to account for the fact that the local angle of attack at the tail is not the same as for the whole aircraft owing to the influence of wing downwash, which increases with wing lift.

Allowing for the downwash angle experienced by the tail, ε , as well as the rigging angle i_t .

$$lpha_t=lpha+i_t-\epsilon$$
 and $rac{\partiallpha_t}{\partiallpha}=1-rac{\partial\epsilon}{\partiallpha}$ (note that rigging angle is not relevant to stability!)

So that
$$\frac{\partial C_{L_t}}{\partial \alpha} = \frac{\partial C_{L_t}}{\partial \alpha_t} \frac{\partial \alpha_t}{\partial \alpha} = \frac{\partial C_{L_t}}{\partial \alpha_t} \left[1 - \frac{\partial \epsilon}{\partial \alpha} \right] \equiv \left(\frac{\partial C_L}{\partial \alpha} \right)_t \left[1 - \frac{\partial \epsilon}{\partial \alpha} \right]$$

A reasonable approximation for the downwash angle if the tail is well downstream of the wing is to use the value of downwash angle far behind an elliptically loaded wing:



$$\epsilon pprox rac{2C_{Lw}}{\pi A}$$
 of

$$\epsilon pprox rac{2C_{Lw}}{\pi A} \quad ext{ or } \quad rac{\partial \epsilon}{\partial lpha} pprox rac{2}{\pi A} rac{\partial C_{Lw}}{\partial lpha}$$

Now from
$$C_L = C_{Lw} + C_{Lt} \frac{S_{ht}}{S}$$
 we

Now from
$$C_L = C_{Lw} + C_{Lt} \frac{S_{ht}}{S}$$
 we have $\frac{\partial C_L}{\partial \alpha} = \left(\frac{\partial C_L}{\partial \alpha}\right)_w + \left(\frac{\partial C_L}{\partial \alpha}\right)_t \left[1 - \frac{\partial \epsilon}{\partial \alpha}\right] \frac{S_{ht}}{S}$

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Neutral Point and the Static Margin

So from
$$(h_{np} - h_{nw}) = V_{ht} \frac{\partial C_{Lt}}{\partial C_L} \equiv V_{ht} \frac{\partial C_{L_t}/\partial \alpha}{\partial C_L/\partial \alpha}$$
 we get $(h_{np} - h_{nw}) = V_{ht} \frac{\left(\frac{\partial C_L}{\partial \alpha}\right)_t \left[1 - \frac{\partial \epsilon}{\partial \alpha}\right]}{\left(\frac{\partial C_L}{\partial \alpha}\right)_w + \left(\frac{\partial C_L}{\partial \alpha}\right)_t \left[1 - \frac{\partial \epsilon}{\partial \alpha}\right] \frac{S_{ht}}{S}}$

$$(h_{np} - h_{nw}) = V_{ht} \frac{\left(\frac{\partial C_L}{\partial \alpha}\right)_t \left[1 - \frac{\partial \epsilon}{\partial \alpha}\right]}{\left(\frac{\partial C_L}{\partial \alpha}\right)_w + \left(\frac{\partial C_L}{\partial \alpha}\right)_t \left[1 - \frac{\partial \epsilon}{\partial \alpha}\right] \frac{S_{ht}}{S}}$$

with, for each lifting surface (*w,t*), $\frac{\partial C_L}{\partial \alpha} \approx \frac{2\pi}{1+2/A}$ where *A* is the aspect ratio of that surface

$$\frac{\partial C_L}{\partial \alpha} \approx \frac{2\pi}{1 + 2/A}$$

and

$$1 - \frac{\partial \epsilon}{\partial \alpha} \approx 1 - \frac{2}{\pi A_w} \frac{\partial C_{L_w}}{\partial \alpha}$$

NOW we have the location for the neutral point purely from aerodynamics.

A little further rearrangement gives

$$\frac{(h_{np} - h_{nw})\bar{c}}{l_{ht}} = \frac{\left(\frac{\partial C_L}{\partial \alpha}\right)_t \left[1 - \frac{\partial \epsilon}{\partial \alpha}\right] \frac{S_{ht}}{S}}{\left(\frac{\partial C_L}{\partial \alpha}\right)_w + \left(\frac{\partial C_L}{\partial \alpha}\right)_t \left[1 - \frac{\partial \epsilon}{\partial \alpha}\right] \frac{S_{ht}}{S}} = \frac{\partial L_t/\partial L_w}{1 + \partial L_t/\partial L_w}$$

$$\frac{\partial L_t}{\partial L_w} = \frac{(\partial C_L/\partial \alpha)_t}{(\partial C_L/\partial \alpha)_w} \left[1 - \frac{\partial \epsilon}{\partial \alpha}\right] \frac{S_{ht}}{S}$$

where $\frac{\partial L_t}{\partial L_w} = \frac{(\partial C_L/\partial \alpha)_t}{(\partial C_L/\partial \alpha)_w} \left[1 - \frac{\partial \epsilon}{\partial \alpha}\right] \frac{S_{ht}}{S}$ is the rate of change of tail lift force w.r.t. wing lift force.

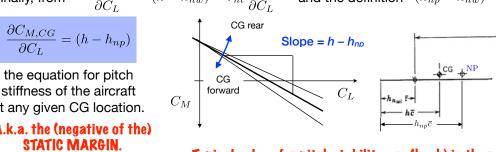
$$\frac{\partial C_{M,CG}}{\partial C_L} = (h - h_{nw}) - V_{ht} \frac{\partial C_{Lt}}{\partial C_L}$$

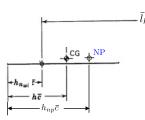
Finally, from
$$\frac{\partial C_{M,CG}}{\partial C_L} = (h-h_{nw}) - V_{ht} \frac{\partial C_{Lt}}{\partial C_L}$$
 and the definition $(h_{np}-h_{nw}) = V_{ht} \frac{\partial C_{Lt}}{\partial C_L}$

$$\frac{\partial C_{M,CG}}{\partial C_L} = (h - h_{np})$$

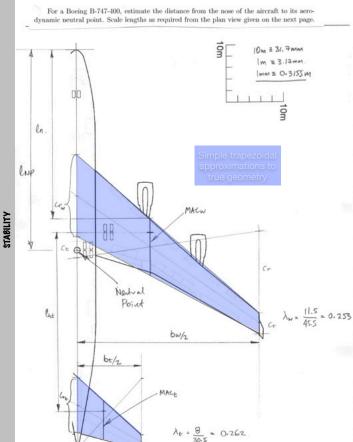
stiffness of the aircraft at any given CG location.

A.k.a. the (negative of the) STATIC MARGIN.





Typical values for pitch stability are (h_n-h) in the range 0.02 - 0.10.



length scale lum = 0.3/55m. See geometric constructions for MAC size & locations of wing I har izon for to I ln = distance from nose to to of MAC 1 = 0.257 = 93.7×0.3/55 m = 29.56 m JOB TITLE Wing geometry Cn = 45.5x 0.3155 m = 14.36m. 6/2 = 100.5 × 0.31514 = 31.7141. S= b C (1+1) = 31.71 x 14.36 x 1.253 m2 = 570.5 m2 $A \cdot b^2 = (2 \times 31.71)^2 = 7.05.$ 2a (1+ h+x²) - 2 x (4.36 (1.+ 0.253+0.255²) m = 10.064 (scaled from drawing, Z = 32x0.355 4 = 10.1 m V). Tail Geometry le from drawing = 99.3×0.355 m= 31.33 m. X = 0.262 Cv = 30.5x 0.3155 m = 9.623 m. b/2 = 35.3x 0.3155 m = 11.14 m. S= 6 Co(1+x)= 0.623x 11.14 x 1.262 m2 = 135.3 m2 $A = 6^2 = \frac{(2 \times 11.14)^2}{135.3} = 3.67$ = 2 Cm (1+1+12) = 2×9-623 (1.262+0.2622) m = 6.76 m She = 135.3 = 0.2372, Vht = Shelk = 0.2372×31.33

