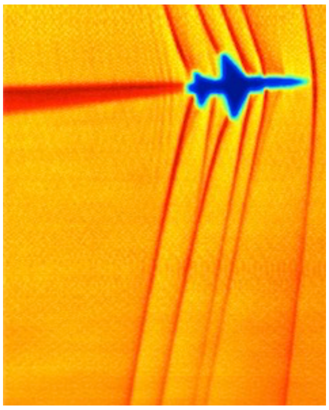


Introduction to compressible flow

Torenbeek & Wittenberg Ch 9
Anderson Chs 5 & 11
Brandt et al. Ch 4.6

Compressible flows

We have so far dealt with fluid flows where the density is taken to be constant within the flow, and these are commonly said to be incompressible.

Typically, incompressible flows are those for which the local velocities are everywhere moderately small compared to the speed of sound, *a*.

$$a = \sqrt{\gamma \mathrm{R} T} = \sqrt{\gamma p / \rho}$$

The speed of sound in liquids is very high compared to gases, so most flows that occur in liquids can be taken as incompressible. In air, the speed of sound at sea level is only 340.3 m/s or 1225 km/hr.

Recall that we defined the Mach number as the ratio of the flow speed to the speed of sound: $M = V/a$. Density variations are usually taken to be significant once speeds reach $M = 0.5$.

Compressible flows have a much richer set of possible behaviours than incompressible ones, and are more difficult to analyse. We must account for internal energy variations in the fluid, which introduces one more differential equation (an energy equation → equation for temperature variation), as well an equation of state that links temperature, pressure and density (e.g. the ideal gas law).

Incompressible Flows

Variables $(\boldsymbol{u}, p)$ (4 if 3D)

Equations needed (4 if 3D):

- Continuity equation
- Momentum equations (3 if 3D)

Compressible Flows

Variables $(\boldsymbol{u}, p, \rho, T)$ (6 if 3D)

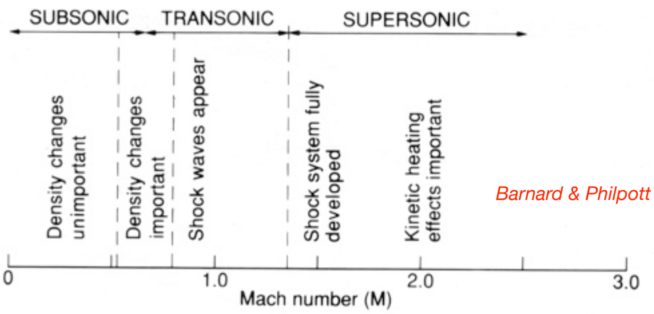
Equations needed (6 if 3D):

- Continuity equation
- Momentum equations (3 if 3D)
- Energy equation
- Equation of state (e.g. ideal gas law)

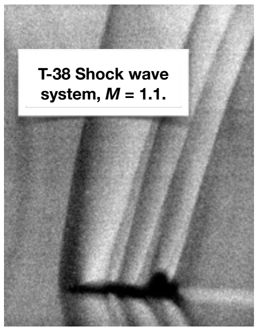
To make a complete study we would need to explain concepts from thermodynamics. Instead we will do what we can with simpler physics, with the occasional introduced result.

Changes arising from compressibility

Nomenclature for free-stream Mach numbers:

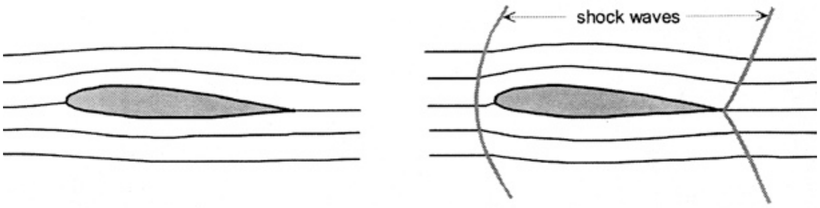


Barnard & Philpott



At subsonic speeds, pressure gradients caused by passage of an aircraft propagate upstream faster than it flies, so streamlines are everywhere smoothly curved.

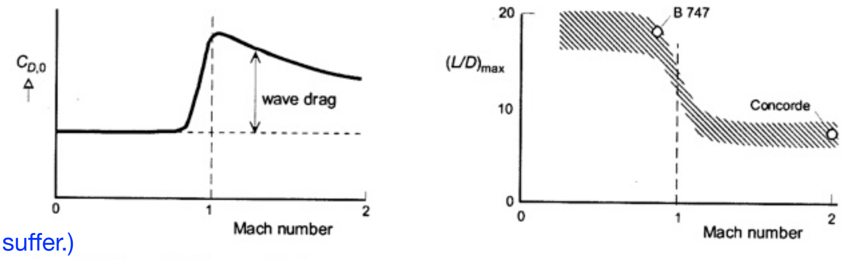
Above supersonic speed, pressure information only travels a limited distance upstream (to a shock wave) where there is an almost discontinuous change in streamline slope.



(a) Subsonic speed (b) Supersonic speed

Shock waves extend a long way from the aircraft and dissipate energy, leading to a new source of drag (*wave drag*). Consequently the best L/D ratio of supersonic aircraft is typically much lower than for subsonic ones.

(NB: for a jet $R \propto M \times L/D$, so range may not suffer.)

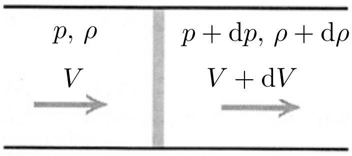


(a) Zero-lift drag coefficient (b) Maximum lift/drag ratio of airlines

Torenbeek & Wittenberg

The speed of sound

Torenbeek & Wittenberg



The speed of sound is the speed of propagation of an infinitesimal pressure variation in a medium.

Consider the propagation of a pressure wave in a duct of constant area.

Continuity (ρVA = const). $\rho V = (\rho + \mathrm{d}\rho)(V + \mathrm{d}V) \quad \rightarrow \quad \rho \mathrm{d}V = -V\mathrm{d}\rho - \mathrm{d}\rho\,\mathrm{d}V$

Momentum (sum of forces = momentum flow out – momentum flow in), ignore any tangential/friction forces.

$$p - (p + \mathrm{d}p) = \frac{\dot{m}}{A}\left[(V + \mathrm{d}V) - V\right] \quad \rightarrow \quad p - (p + \mathrm{d}p) = (\rho + \mathrm{d}\rho)(V + \mathrm{d}V)^2 - \rho V^2$$

Substituting continuity into momentum (twice) we find

$$\frac{\mathrm{d}p}{\mathrm{d}\rho} = V^2\left(1 + \frac{\mathrm{d}V}{V}\right)$$

For an infinitesimally small disturbance, $\mathrm{d}V/V \rightarrow 0$ and we have the speed of sound $\frac{\mathrm{d}p}{\mathrm{d}\rho} = a^2$

To make further progress we need to introduce (thermodynamics) isentropic relationship for an ideal gas

$\frac{\mathrm{d}p}{\mathrm{d}\rho} = \frac{\gamma p}{\rho}$ using the ratio of specific heats $\gamma = C_p/C_v$ with gas constant $\mathrm{R} = C_p - C_v$

So that (as we introduced earlier) $a = \sqrt{\frac{\gamma p}{\rho}} = \sqrt{\gamma \mathrm{R}T}$

For air at moderate temperatures

$\gamma = 1.40$

$\mathrm{R} = 287.05$ J/kg.K

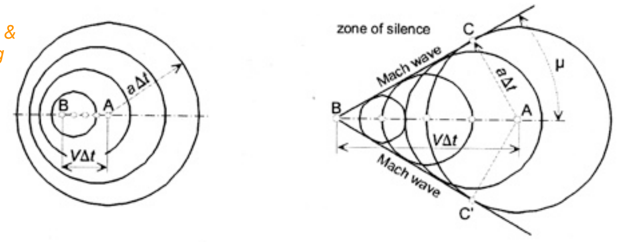
Simple to show that $\frac{1}{2}\rho V^2 = \frac{1}{2}\gamma p M^2$

Pressure propagation and Mach waves

Many of the observed consequences of compressible flows stem from the fact that the speed of sound (propagation of pressure waves) is finite.

Sound (or pressure information) from a source propagates in spherical fronts at speed *a*, but if the object is also moving at speed *V*, the source also moves. If the speed $V > a$, the fronts form a *Mach cone*, and in front of the cone (the *zone of silence*) the object cannot be heard.

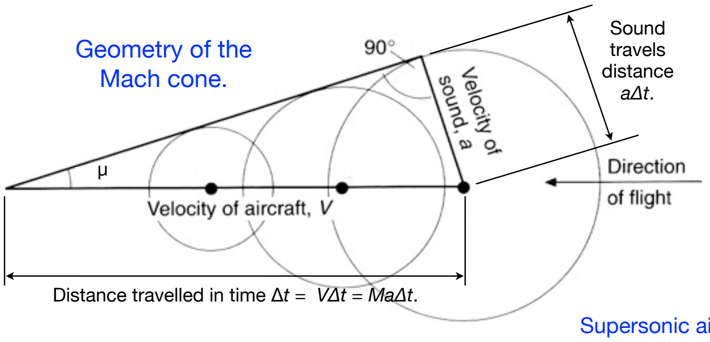
Torenbeek & Wittenberg



(a) Subsonic speed ($V < a$, $M < 1$) (b) Supersonic speed ($V > a$, $M > 1$)

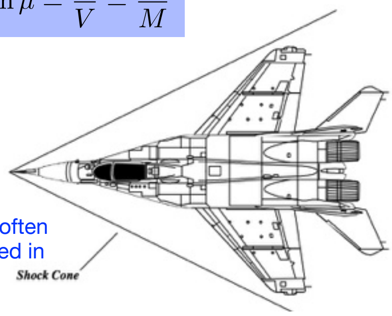
The wave front of the cone is called a *Mach wave* or *shock wave.*

Geometry of the Mach cone.



From elementary geometry we find that

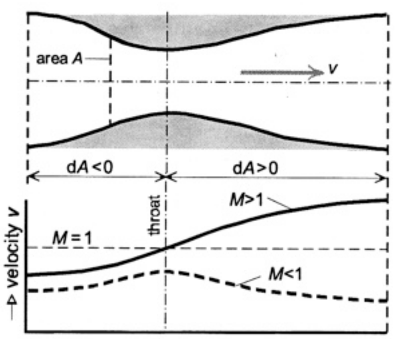
$$\sin\mu = \frac{a}{V} = \frac{1}{M}$$



Supersonic aircraft are often designed to fly contained in their Mach cone.

Flow in a channel

Analysis of 1D flow in a channel of variable area provides some perhaps counter-intuitive outcomes.



Torenbeek & Wittenberg

Using continuity $\rho V A = \text{const.}$ and *logarithmic differentiation* $[\ln(f)]' = \frac{f'}{f}$

$$\frac{\mathrm{d}\rho}{\rho} + \frac{\mathrm{d}V}{V} + \frac{\mathrm{d}A}{A} = 0$$

$$\frac{\mathrm{d}\rho}{\mathrm{d}p}\frac{\mathrm{d}p}{\rho} + \frac{\mathrm{d}V}{V} + \frac{\mathrm{d}A}{A} = 0$$

Now $\frac{\mathrm{d}p}{\mathrm{d}\rho} = a^2$ and (Euler's equation) $\mathrm{d}p = -\rho V \mathrm{d}V$

Rearranging $-\frac{1}{a^2}V\mathrm{d}V + \frac{\mathrm{d}V}{V} + \frac{\mathrm{d}A}{A} = 0 \rightarrow (1 - M^2)\frac{\mathrm{d}V}{V} + \frac{\mathrm{d}A}{A} = 0 \rightarrow (M^2 - 1)\frac{\mathrm{d}V}{V} = \frac{\mathrm{d}A}{A}$

If flow is subsonic ($M < 1$) then flow speeds up ($\mathrm{d}V > 0$) in a converging duct ($\mathrm{d}A < 0$) and slows down in a diverging duct. This is our everyday (subsonic) experience.

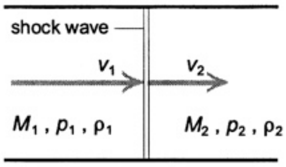
However, the opposite is true if the flow is supersonic: a supersonic flow speeds up in a *diverging* duct and slows down in a converging one. This is brought about by density variation: $\mathrm{d}\rho < 0$ in a diverging supersonic flow.

Finally, the only place we can have $M = 1$ in this flow is the location where $\mathrm{d}A = 0$, i.e. at the throat.

	Subsonic flow ($M < 1$)	*Supersonic flow* ($M > 1$)
converging channel ($\mathrm{d}A < 0$)	$\mathrm{d}v > 0$ $\mathrm{d}p < 0$ $\mathrm{d}T < 0$ $\mathrm{d}\rho < 0$	$\mathrm{d}v < 0$ $\mathrm{d}p > 0$ $\mathrm{d}T > 0$ $\mathrm{d}\rho > 0$
diverging channel ($\mathrm{d}A > 0$)	$\mathrm{d}v < 0$ $\mathrm{d}p > 0$ $\mathrm{d}T > 0$ $\mathrm{d}\rho > 0$	$\mathrm{d}v > 0$ $\mathrm{d}p < 0$ $\mathrm{d}T < 0$ $\mathrm{d}\rho < 0$

Torenbeek & Wittenberg

Normal and oblique shocks waves



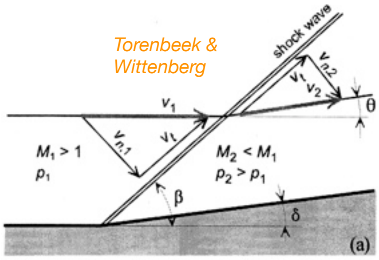
Normal (perpendicular) shock waves can be dealt with using 1D methods. Various outcomes may be derived for relations across normal shocks, e.g.

$$\frac{M_2}{M_1} = \frac{p_1}{p_2}\sqrt{\frac{T_2}{T_1}}$$

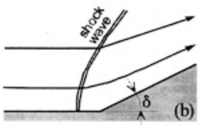
A fundamental result is that in a real fluid, one can only have $M_1 > 1 \text{ and } M_2 < 1$ across a normal shock.

Flow downstream of a normal shock is always subsonic and, with that, leads to a rise in pressure, density, and temperature.

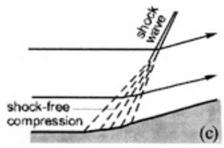
Oblique shocks are a 2D phenomenon caused by a compressive turning. Flow downstream may also be supersonic, but is of lower Mach number than upstream.



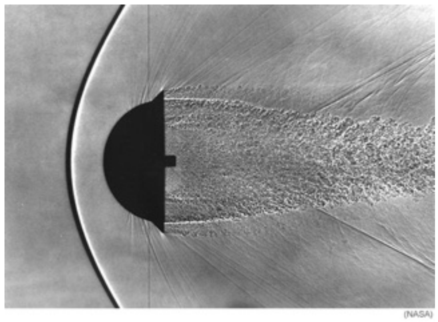
(a) Attached oblique shock wave at a sharp corner with small degree of turn



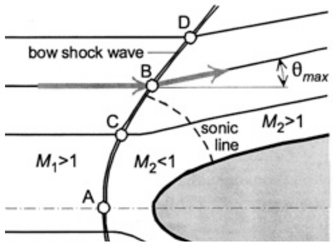
(b) Detached shock wave at a large degree of turn



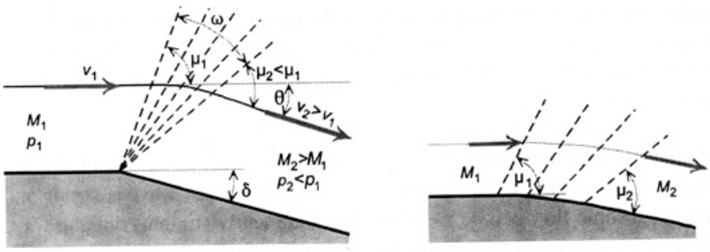
(c) Lambda-shock wave at a gradual turn



Bow shocks ahead of blunt objects (e.g at the LE of an airfoil) are locally normal shocks, blending to oblique further from the body.



Expansion fans, airfoil flows



(a) Sharp deflection

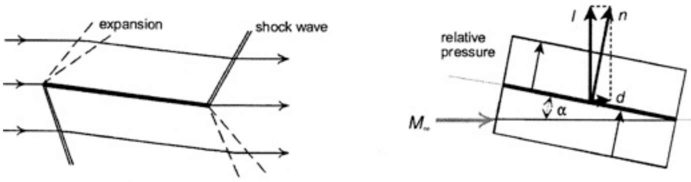
(b) Smooth deflection

Torenbeek & Wittenberg

At an expansion in a 2D supersonic flow, flow speeds up (as it does in 1D) but relatively rapidly in a narrow *expansion fan*.

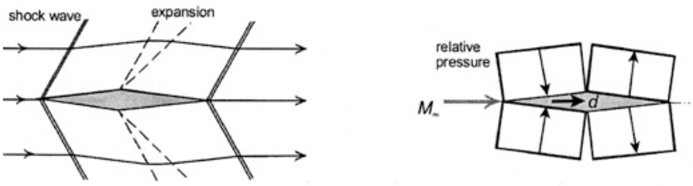
Supersonic flows past airfoils are conceived as a series of sharp turns generated by oblique shock waves and expansion fans.

A flat plate airfoil can generate substantial lift owing the resulting pressure difference.



(a) Flat plate at angle of attack

Note that the centre of pressure (and the aerodynamic centre) for such a flow is at 0.5*c*. This is substantially different from the case in subsonic flow (0.25*c*).



(b) Diamond section, zero angle of attack

Owing to the nature of the pressure differences, a non-lifting airfoil of finite thickness generates significant pressure drag (wave drag). Again this is substantially different to subsonic flow.

The wave drag coefficient is proportional to the square of the relative thickness, $(t/c)^2$, so supersonic airfoils are generally quite thin.

COMPRESSIBLE FLOW

Transonic flows past airfoils

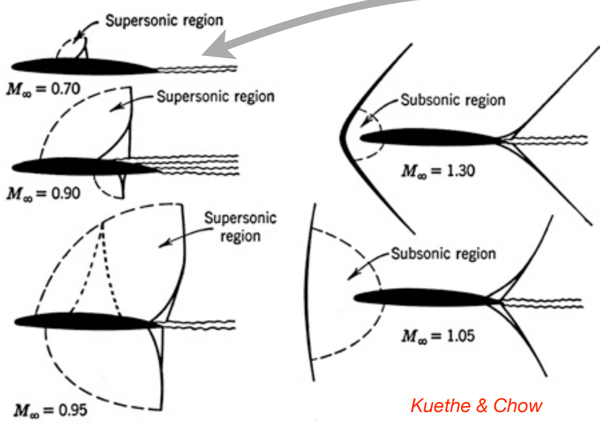
The general progression of airfoil shock structure with free-stream Mach number:

The *critical* free-stream Mach number M_{cr} is the value for which the flow around the body (airfoil) first reaches $M = 1$ locally.

Local supersonic flow and a compression shock arise at a 'critical' freestream $M_\infty<1$.

For transonic flows, $M_\infty<1$, the shock can produce flow separation and greatly increased drag.

Supersonic region grows to include entire upper and lower surfaces. Compression shock moves to TE.

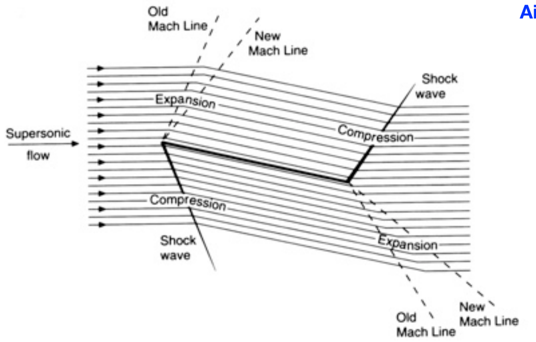


Kuethe & Chow

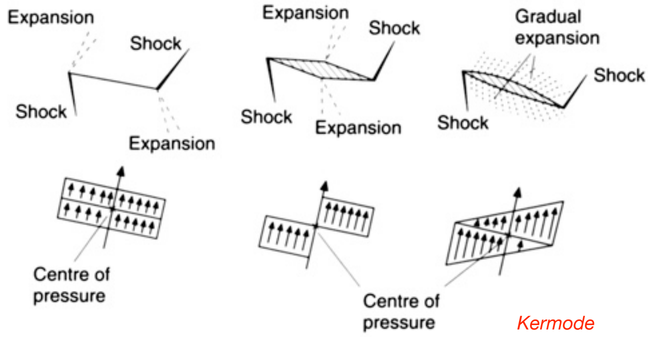
Generally, much less separation for supersonic flows but wave drag becomes dominant.

Bow shock appears at $M_\infty = 1$.

In supersonic flows, lift is mainly generated by sudden compression (shocks) and expansions.



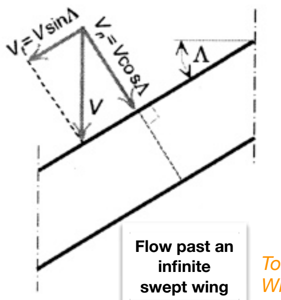
Airfoil type: Flat plate, Diamond, Curved



Kermode

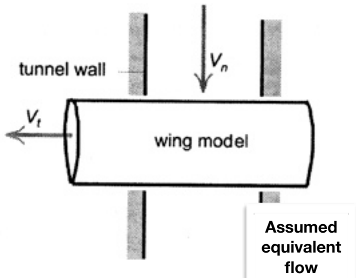
COMPRESSIBLE FLOW

Wing sweep and transonic flow



Torenbeek & Wittenberg

(a) Resolution of the flow speed



(b) Sideways sliding model in a wind tunnel

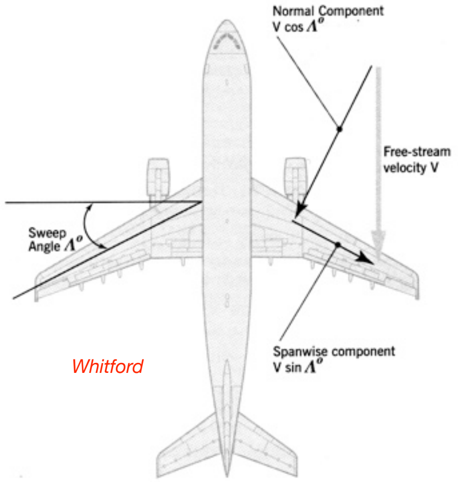
Assumption: only the component of a flow normal to the wing is responsible for generating pressure variations around the airfoil.

By sweeping the wing back (or forward) through angle Λ, the velocity component normal to the wing is reduced to $V \cos\Lambda$

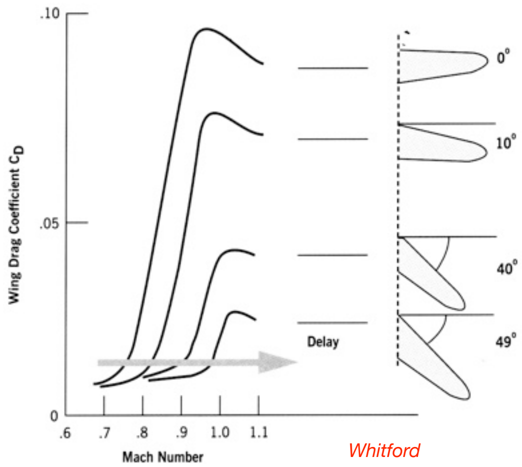
$$(M_{\text{crit}})_\Lambda = \frac{(M_{\text{crit}})_{\Lambda=0}}{\cos\Lambda}$$

Consequently, compressibility effects in transonic flow can be delayed to higher freestream Mach numbers by introducing wing sweep.

Compressibility effects are dominated by flow normal to wing LE, and are reduced with wing sweep.



Whitford



Whitford