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Initial sizing+constraint analysis – Example

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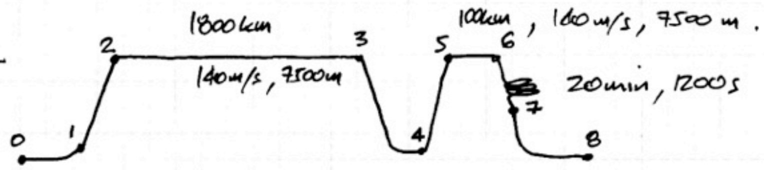
Initial sizing of a twin-engine turboprop passenger aircraft, 12 PAX

Top-Level Requirements

Payload	1 crew + 10kg, 12 PAX @ 82kg ea. + 20kg baggage,
Range	1800 km @ max payload
Reserve fuel	10 km flight to alternate + 20 min loiter
Maximum cruise speed/altitude	140 m/s @ 7500 m
Minimum service ceiling	8500 m
Max TOFL @ MTOW, 1km altitude	1000 m
Max landing approach speed, MTOW, 1km.	55 m/s (1.3 V_{STALL})
Minimum climb gradient, OEI, 1km alt.	1.5% (1.2 V_{STALL})

Concept: Conventional, tractor engines, low wing, Pressurized cabin
Tricycle landing gear, retract.
Minimal/no wing sweep, subsonic operation (140 m/s ~ 0.5M)

Mission



ISA data

h(m)	σ	ρ (kg/m³)	a (m/s)
0	1	1.225	340.3
1000	0.9075	1.112	336.5
3750	0.6861	0.8405	325.6
7500	0.4538	0.5559	310.1
8500	0.4047	0.4958	306.0

JOB TITLE

Engine performance model

Reasonable approximations
$$\alpha_{climb} = \frac{P_{es}}{P_{es0}} = 0.9\sigma^{0.8}$$
$$\alpha_{cruise} = 0.7\sigma^{0.8}$$

Constant-speed prop, $\eta_{pr} = 0.85$

Raymer $\begin{cases} C = 0.085 \text{ mg/W.s} & \text{cruise \&} \\ \;\;\, = 0.101 \text{ mg/Ws} & \text{loiter} \end{cases}$

Available/typical data for regional turboprops (Schaufele)

$C_{D,0} \sim 0.02$ (+ Torenbeek)

W_0/S in range 1900 – 4300 Pa

$C_{L\,max}$ clean 1.5 – 1.8

$C_{L\,max}$ TO 1.7 – 2.2

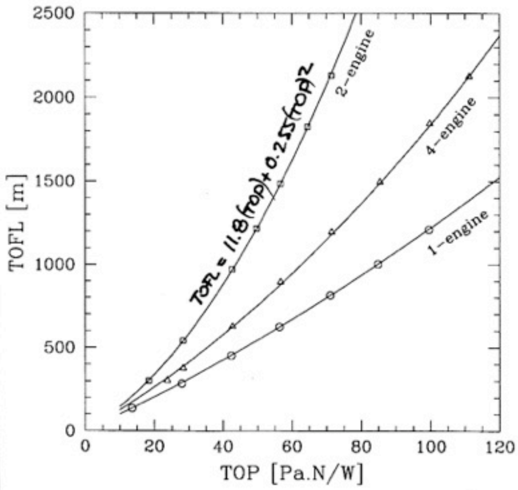
$C_{L\,max}$ Land 1.9 – 2.7

Wing aspect ratio 11.0 – 12.8
taper 0.4 – 0.6
t/c 0.12 – 0.15

TOFL correlation for 2 piston/prop engines

where TOP $= \frac{W_0}{S}\frac{W_0}{P}\frac{1}{\sigma}\frac{1}{C_{L\,max}}$ $\frac{\text{Pa.N}}{\text{W}}$:

TOFL (m) = 11.8 TOP + 0.255 TOP^2



Aerodynamic parameters

Adopt typical values for class $C_{D,0} = 0.02$, $A = 11$, $e = 0.80$

Hence estimate $K = \frac{1}{\pi A e} = \frac{1}{\pi \times 11 \times 0.8} = \frac{1}{27.65} = \underline{0.036}$

$$\left(\frac{L}{D}\right)^* = \frac{1}{\sqrt{4C_{D0}K}} = \frac{1}{\sqrt{4\times0.02\times0.036}} = \underline{18.6}$$

$$(C_L)^* = \sqrt{\frac{C_{D0}}{K}} = \sqrt{\frac{0.02}{0.036}} = \underline{0.745}.$$

$C_{L\,max}$: typical values are $C_{L\,max\,clean} = 1.6$

$C_{L\,max\,TO} = 2.0$

$C_{L\,max\,land} = 2.3$.

Initial weight estimate

We will adopt Raymer's correlation for class

Very approximately, $\frac{W_e}{W_0} \sim 0.6$ (Raymer)

$$\frac{W_e}{W_0} \doteq 0.92\, M_0^{-0.05}, \quad M_0 = \frac{W_0}{g} \text{ in kg}.$$

NB Raymer has W_0 here, should be $\frac{W_0}{g}$.

Historical/statistical values only (Raymer):

$$\frac{W_1}{W_0} = 0.970$$
$$\frac{W_2}{W_1} = 0.985$$
$$\frac{W_4}{W_3} = 0.995$$
$$\frac{W_5}{W_4} = 0.985$$
$$\frac{W_8}{W_7} = 0.995.$$

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Cruise $\sim (L/D)^*$ $\quad \dfrac{W_i}{W_{i-1}} = \exp\left(\dfrac{-R g c}{\eta_{pr} (L/D)^*}\right)$ does not involve speed

Loiter $\sim 0.866 (L/D)^*$ $\quad \dfrac{W_i}{W_{i-1}} = \exp\left(\dfrac{-E g c \times 0.760 V^*}{\eta_{pr} \times 0.866 (L/D)^*}\right)$ does involve speed

Now, the loiter speed and height were not supplied. Assume its flown at mid altitude (3750m) where $\rho = 0.8405$ kg/m³

Assume an approximate wing loading (base) of $W_0/S \sim 3$ kPa and use $\beta \sim 0.7$

Hence $\underline{V^*_{loiter}} = \sqrt{\dfrac{2}{\rho}\beta\dfrac{W_0}{S}\dfrac{1}{C_L^*}} = \sqrt{\dfrac{2}{0.8405} \times \dfrac{3\times10^3 \times 0.7}{0.745}}$ m/s $= \underline{81.9 \text{ m/s}}$

Hence $\dfrac{W_3}{W_2} = \exp\left(-\dfrac{1800\times10^3 \times 9.81 \times 0.085\times10^{-6}}{0.85 \times 18.6}\right) = 0.909$

$\dfrac{W_6}{W_5} = \exp\left(-\dfrac{100\times10^3 \times 9.81 \times 0.085\times10^{-6}}{0.85 \times 18.6}\right) = 0.995$

$\dfrac{W_7}{W_6} = \exp\left(-\dfrac{1200 \times 9.81 \times 0.101\times10^{-6} \times 0.76 \times 81.9}{0.85 \times 0.866 \times 18.6}\right) = 0.995$

So $\dfrac{W_8}{W_0} = \dfrac{W_1}{W_0} \times \cdots \dfrac{W_8}{W_7} = 0.970 \times 0.985 \times 0.909 \times 0.995 \times 0.985 \times 0.995 \times 0.995 \times 0.995$

$= 0.8385$

Allow a further 6% margin for safety and trapped fuel, $\dfrac{W_f}{W_0} = 1.06(1-0.8385) = \underline{0.1712}$

JOB TITLE

6

Payload PAX & crew @ 82 kg each

~~Baggage~~ $12\times20 + 1\times10$ kg.

M Payload $= 13\times82 + 12\times20 + 1\times10$ kg $= \underline{1316 \text{ kg}}$

MTOW estimate. $W_0 = \dfrac{W_{payload}}{1-\frac{W_f}{W_0}-\frac{W_e}{W_0}}$, $m_0 = \dfrac{m_{payload}}{1-0.1712-0.92 m_0^{-0.05}}$

Solve Iteratively

m_0 kg	RHS kg
5000	5776
5776	5668
5668	5682
5682	5680

$\underline{m_0 = 5680 \text{ kg}}$

$\underline{W_0} = g m_0 = 9.81 \times 5680$ N $= \underline{55.72 \text{ kN}}$

And $\dfrac{W_e}{W_0} = 0.92 \times 5680^{-0.05} = 0.597 \triangleq 60\%$ (typical of class).

$W_f = 0.1712 \times 5680 \text{ kg} \times 9.81 = 9.54$ kN, 972 kg.

(Density of JET-A1 fuel is 0.80 kg/L

∴ Fuel tank capacity $\triangleq$ 1216 L, 1.216 m³).

JOB TITLE

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Constraint Analysis. Rubber-engine sizing

JOB TITLE

Landing Approach speed $1.3V_{STALL} = 55\ m/s$ @ MTOW, $\beta = 1$, 1km altitude

$$V_{app} = 1.3V_{STALL} = 1.3\sqrt{\frac{2}{\sigma\rho_0}\beta\frac{W_0}{S}\frac{1}{C_{Lmax_{LAND}}}} \quad \text{or} \quad \frac{W_0}{S} = \frac{C_{Lmax}}{\beta}\frac{\sigma\rho_0}{2}\left(\frac{V_{APP}}{1.3}\right)^2$$

$$\frac{W_0}{S} = \left(\frac{2.3}{1}\right)\times\frac{1.112}{2}\times\left(\frac{55}{1.3}\right)^2 \text{ Pa} = \underline{2289\text{ Pa}}$$ (This is at the low end of the quoted range 1900–4300 Pa, but OK)

Take-off field length

$$\underset{(prop)}{TOP} = \frac{W_0}{S}\frac{W_0}{P_0}\frac{1}{\sigma}\frac{1}{C_{Lmax_{TO}}} \qquad \sigma = 0.9075, \quad C_{Lmax} = 2.$$

Correlation

$$TOFL = 11.8\,TOP + 0.255\,TOP^2 \quad m$$

$$= \frac{11.8}{\sigma C_{Lmax}}\frac{W_0}{S}\frac{W_0}{P_0} + \frac{0.255}{\sigma^2 C_{Lmax}^2}\left(\frac{W_0}{S}\right)^2\left(\frac{W_0}{P_0}\right)^2$$

$$= \frac{11.8}{0.9075\times2}\frac{W_0}{S}\frac{W_0}{P_0} + \frac{0.255}{0.9075^2\times4}\left(\frac{W_0}{S}\right)^2\left(\frac{W_0}{P_0}\right)^2$$

$$1000 = 6.501\,\frac{W_0}{S}\frac{W_0}{P_0} + 77.41\times10^{-3}\left(\frac{W_0}{S}\right)^2\left(\frac{W_0}{P_0}\right)^2 \quad m$$

Clearly this will represent a curve relating $\frac{P_0}{W_0}$ to $\frac{W_0}{S}$.

Take a suitable range of $\frac{W_0}{S}$ values and solve for $\frac{P_0}{W_0}$.
(say 1000 – 4000 Pa in steps of 250 Pa)

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JOB TITLE

TOFL,

$\frac{W_0}{S}$ (Pa)	$\frac{P_0}{W_0}$ (m/s)
1000	12.63
1250	15.79
1500	18.95
1750	22.10
2000	25.26
2250	28.42
2500	31.58
2750	34.73
3000	37.89
3250	41.05
3500	44.21
3750	47.36
4000	50.52

Just as a check, take $\frac{P_0}{W_0} = 12.63\ m/s$, $\frac{W_0}{S} = 1000$ Pa

$\Rightarrow$ TOP = 43.62 PaN/W
$\Rightarrow$ TOFL = 1000 m ✓

Cruise speed Sometimes called "payload-range" constraint

140 m/s @ 7500 m, $\sigma = 0.4538$, $\rho = 0.5559\ kg/m^3$

$$\alpha = \frac{P}{P_0} = 0.7\,\sigma^{0.8} = 0.7\times0.4538^{0.8} = 0.3720$$

Take this at mid-cruise weight, $\beta = \frac{W_1}{W_0}\times\frac{W_2}{W_1}\times\frac{(1+W_3/W_2)}{2}$

$$= 0.970\times0.985\times1.909/2 = \underline{0.912}.$$

$$q = \tfrac{1}{2}\rho V^2 = \tfrac{1}{2}\times0.5559\times140^2 \text{ Pa} = \underline{5448\text{ Pa}}$$

$$\beta W_0 = \tfrac{1}{2}\rho V^2 S C_L = q S C_L \qquad C_L = \frac{\beta}{q}\frac{W_0}{S} = \frac{0.912}{5448}\frac{W_0}{S} = 167.4\times10^{-6}\frac{W_0}{S}.$$

– It is useful to keep C_L explicitly for this constraint as it determines L/D and hence fuel consumption.

The master constraint equation for Piston/Prop-type aircraft is

$$\frac{P_0}{W_0} = \frac{\beta}{\eta_{pr}\alpha}\left\{ V\left[\frac{q}{\beta(W_0/S)} C_{D0} + n^2\frac{\beta(W_0/S)}{q}K\right] + P_s\right\}, \text{ here } n = 1,\ P_s = 0$$

$$= \frac{\beta}{\eta_{pr}\alpha}\left\{ V\left[\frac{C_{D0}}{C_L} + KC_L\right]\right\} = \frac{0.912\times 140}{0.85\times 0.3720}\left[\frac{C_{D0}}{C_L} + KC_L\right]\frac{W}{N},\frac{m}{s}$$

$$= 403\left[\frac{0.02}{C_L} + 0.036\,C_L\right]\frac{W}{N}\left(\frac{m}{s}\right).$$

W_0/S (Pa)	C_L	P_0/W_0 (m/s)
1000	0.1674	50.58
1250	0.2093	41.55
1500	0.2511	35.74
1750	0.2930	31.76
2000	0.3348	28.93
2250	0.3767	26.86
2500	0.4185	25.33
2750	0.4604	24.19
3000	0.5022	23.34
3250	0.5441	22.71
3500	0.5859	22.26
3750	0.6278	21.95
4000	0.6696	21.75
5000	0.8370	21.77

← cf. $C_L^* = 0.745$

Straight away we have an indication of a difficulty: the landing approach speed constraint limits $\frac{W_0}{S} < 2289$ Pa. Hence our cruise lift coefficient is going to be a long way from the optimal value of C_L^*.

JOB TITLE

Service Ceiling 8500 m. $V_{V_{max}, min} = 100$ ft/min $= 0.508$ m/s.

For a prop aircraft $V_{V_{max}} = \eta_{pr}\frac{\alpha}{\beta}\frac{P_0}{W_0} - \frac{2}{\rho}\sqrt{\frac{K}{3C_{D,0}}}\left(\frac{W_0}{S}\right)^{1/2}\frac{1.155\beta^{1/2}}{(L/D)^*}$

$$\text{or } \frac{P_0}{W_0} \geqslant \frac{\beta}{\alpha}\frac{V_{V_{max}}}{\eta_{pr}} + \frac{2\beta^{3/2}}{\eta_{pr}\alpha\sigma\rho_0}\sqrt{\frac{K}{3C_{D,0}}}\left(\frac{W_0}{S}\right)^{1/2}\frac{1.155}{(L/D)^*}$$

$\alpha = \alpha_{climb} = 0.9\sigma^{0.8}$, $\sigma = 0.4047$, $\rho = 0.4958$ kg/m^3

$= 0.9 \times 0.4047^{0.8}$ Say $\beta = \frac{W_1}{W_0}\times\frac{W_2}{W_1} = 0.970\times 0.985$

$= 0.4365.$ $= 0.9555.$

$$\frac{P_0}{W_0} \geqslant \frac{0.9555}{0.4365}\times\frac{0.508}{0.85} + \frac{2\times 0.9555^{3/2}}{0.85\times 0.4365\times 0.4958}\sqrt{\frac{0.036}{3\times 0.02}}\times\frac{1.155}{18.6}\sqrt{\frac{W_0}{S}}$$

$$\geqslant 1.308 + 0.4884\sqrt{W_0/S}.$$

W_0/S (Pa)	P_0/W_0 (m/s).
1000	16.75
1250	18.58
1500	20.22
1750	21.74
2000	23.15
2250	24.47
2500	25.73
2750	26.92
3000	28.06
3250	29.15
3500	30.20
3750	31.22
4000	32.20

JOB TITLE

Climb Gradient 1.5% @ 1km elevation airstrip, One engine inoperative.

$$\tan\theta \simeq \sin\theta = \frac{P - DV}{WV} = \frac{\alpha \eta_{pr} P_0}{\beta W_0 V_2} - \frac{D}{\beta W_0} = 0.015$$

$V = V_2 = 1.2\, V_{STALL\,TO}$ we know $C_L = \frac{C_{L\,STALL}}{1.2^2} = \frac{2.0}{1.44} = 1.389.$

$$D = \tfrac{1}{2}\rho V^2 S (C_{D0} + K C_L^2) = qS(0.02 + 0.036 \times 1.389^2) = qS \times \underbrace{0.0894}_{C_D}.$$

$$\frac{D}{S} = C_D q.$$

$$\sin\theta = \left(\frac{\alpha}{\beta}\frac{\eta_{pr}}{V_2}\right)\frac{P_0}{W_0} - \frac{1}{\beta}\frac{D}{S}\left(\frac{W_0}{S}\right)^{-1} = \left(\frac{\alpha}{\beta}\frac{\eta_{pr}}{V_2}\right)\frac{P_0}{W_0} - \frac{1}{\beta}\frac{q\,C_D}{(W_0/S)}.$$

$$\alpha = \alpha_{max} = \sigma^{0.8} = 0.9075^{0.8} = 0.9253. \qquad \beta = 1.$$

$$V_2 = 1.2\, V_{STALL} = 1.2\sqrt{\frac{2}{\sigma\rho_0}\beta\frac{W_0}{S}\frac{1}{C_{Lmax}}} = \left(\frac{1.44 \times 2 \times \beta}{\sigma\rho_0\, C_{Lmax}}\right)^{1/2}\left(\frac{W_0}{S}\right)^{1/2}$$

$$= \left(\frac{1.44 \times 2 \times 1}{1.112 \times 2.0}\right)^{1/2}\left(\frac{W_0}{S}\right)^{1/2} = 1.138\left(\frac{W_0}{S}\right)^{1/2}.$$

$$q = \tfrac{1}{2}\rho V_2^2 = \frac{\cancel{\rho}}{\cancel{2}} \times 1.44 \frac{\cancel{2}}{\cancel{\rho}}\frac{\beta}{C_{Lmax}}\left(\frac{W_0}{S}\right) = \frac{1.44}{2}\left(\frac{W_0}{S}\right)$$

So

$$\sin\theta \simeq \tan\theta = \left(\frac{\alpha}{\beta}\frac{\eta_{pr}}{1.138(W_0/S)^{1/2}}\right)\frac{P_0}{W_0} - \frac{1}{\beta}\frac{\cancel{(W_0/S)}C_D}{\cancel{(W_0/S)}} \times \frac{1.44}{C_{Lmax}}$$

$$= \left(\frac{\alpha}{\beta}\frac{\eta_{pr}}{1.138}\right)\left(\frac{P_0}{W_0}\right)\left(\frac{W_0}{S}\right)^{-1/2} - \frac{1.44}{\beta C_{Lmax}} C_D.$$

or

$$\left(0.015 + \frac{1.44}{\beta C_{Lmax}} C_D\right) = \left(\frac{\alpha}{\beta}\right)\left(\frac{\eta_{pr}}{1.138}\right)\left(\frac{P_0}{W_0}\right)\frac{1}{(W_0/S)^{1/2}}$$

$$\frac{P_0}{W_0} = \frac{\left(0.015 + \frac{1.44 C_D}{\beta C_{Lmax}}\right)\left(\frac{W_0}{S}\right)^{1/2}}{\left(\frac{\alpha}{\beta}\frac{\eta_{pr}}{1.138}\right)} = \frac{\left(0.015 + \frac{1.44 \times 0.0894}{1 \times 2.0}\right)}{\left(\frac{0.9253}{1} \times \frac{0.85}{1.138}\right)}\left(\frac{W_0}{S}\right)^{1/2}$$

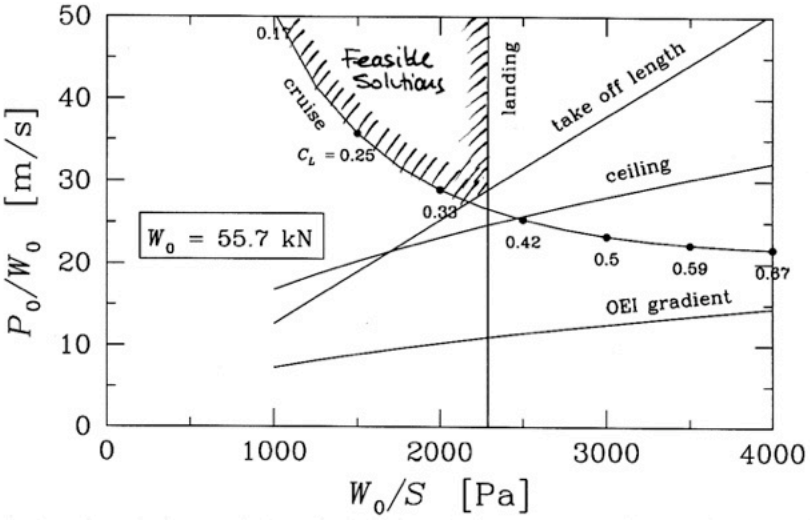
$= 0.1148\left(\frac{W_0}{S}\right)^{1/2}$. This is the required power on one engine. Installed power must be twice this

$$\therefore \frac{P_0}{W_0} = 2 \times 0.1148\sqrt{\frac{W_0}{S}} = 0.2296\left(\frac{W_0}{S}\right)^{1/2}$$

W_0/S (Pa)	P_0/W_0 (W/S)
1000	7.261
1250	8.118
1500	8.892
1750	9.605
2000	10.27
2250	10.89
2500	11.48
2750	12.04
3000	12.58
3250	13.09
3500	13.58
3750	14.06
4000	14.52

That completes all the required performance constraints and we plot them on a single graph.

Initial constraint Plot



We see that the three operative constraints (with our present aerodynamic choices of C_{D0} and K) are

1 Landing
2 Takeoff field length
3 Cruise at given speed altitude.

Remarks:

a) We could influence constraint 1 & 2 in our favour by using a higher-performance high-lift system or high-lift airfoil section. We would certainly want to review this with care. However for a small aircraft an expensive system may not be warranted.

b) The TOFL constraint is based on a (rather old) correlation. We'd want to carefully review this.

c) Even more significantly: our cruise C_L is nowhere near C_L^*. This will mean more fuel use than we'd envisaged.

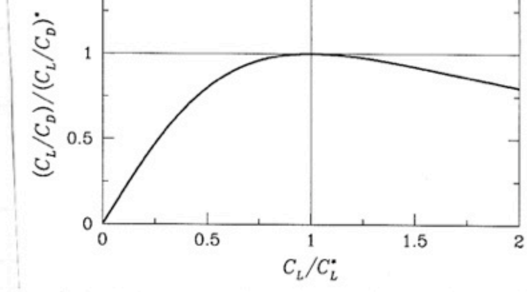
(It is extremely common that prop aircraft are called on to cruise faster than their best L/D speed – largely because that's typically quite low).

d) We could also review the design constraints to check the assumptions (e.g. that Vapp is @ MTOW). Client liaison?

For now let's go back and review our fuel use / MTOW estimate based on cruising at

$$\frac{W_0}{S} = 2250 \text{ Pa where } C_L = 0.3767$$

This is much lower than $C_L^* = 0.745$ but the impact is not quite as severe as we may initially expect, owing to the shape of the relationship between C_L/C_L^* and $L/D/(L/D)^*$:



$$L/D = \frac{1}{C_{D0}/C_L + KC_L}$$

$$= \frac{1}{0.02/0.3767 + 0.036 \times 0.3767}$$

$$= \underline{15.0} \quad (\text{cf } (L/D)^* = 18.6).$$

Now

$$\frac{W_3}{W_2} = \exp\left(-\frac{Rgc}{\eta_{pr} L/D}\right)$$

$$= \exp\left(-\frac{1800\times10^3 \times 9.81 \times 0.085\times10^{-6}}{0.85\times 15.0}\right)$$

$$= \underline{0.889} \quad (\text{cf } 0.909)$$

And

$$\frac{W_6}{W_5} = \exp\left(\frac{-100\times10^3 \times 9.81 \times 0.085\times10^{-6}}{0.85\times15.0}\right)$$

$$= \underline{0.993} \quad (\text{cf } 0.995)$$

Changes are not large but will have a significant impact on W_0.

$$\frac{W_8}{W_0} = 0.970 \times 0.985 \times 0.889 \times 0.995 \times 0.985 \times 0.993 \times 0.995 \times 0.995$$
$$= 0.8143 \quad (\text{cf } 0.8385 \text{ before.})$$

Allow a further 6% margin : $\frac{W_f}{W_0} = 1.06\left(1 - 0.8143\right) = 0.1968$.

$$W_0 = \frac{1316}{1 - 0.1968 - 0.92 W_0^{-0.05}}.$$

M_0 (kg)	RHS (kg)
5700	6383
6380	6280
6280	6295
6295	6293

say 6295 kg

(11% increase.)

So our new estimate of $W_0 = 9.81 \times 6295$ N $= 61.75$ kN

This is a reasonable point to consider the available powerplant. Pratt & Whitney Canada turn out a range of suitable turboprop engines. After some analysis of their offerings :

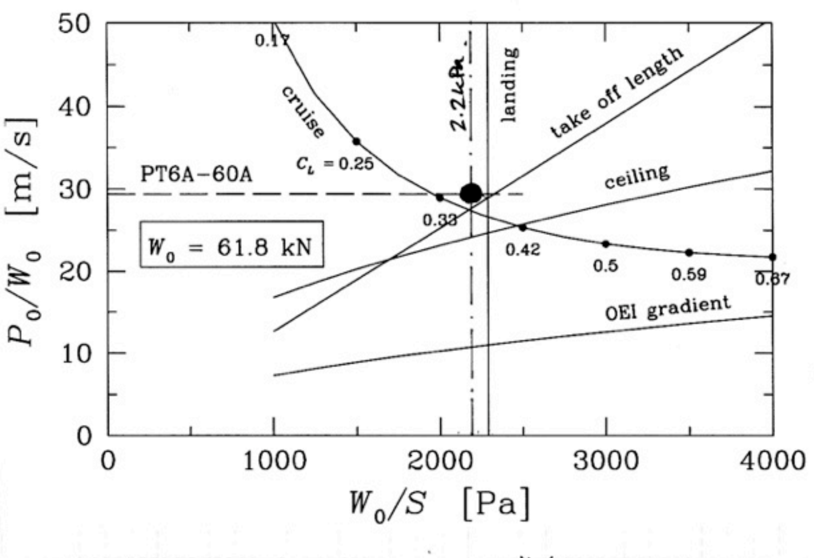
PT6A-60A provides 1218 ESHP, @ 1700 rpm.

Gives an installed equivalent shaft power of $2 \times 1218 \times 745.7$ W $= 1.817$ MW.

Hence $\frac{P_0}{W_0} = \frac{1.817 \times 10^6}{61.75 \times 10^3}$ m/s $= 29.4$ m/s

This lies right near the value at the intersection of the TOFL and landing constraints.

Revised Constraint plot, Suitable Powerplant shown.



This looks reasonable - some power margin available, and the TOFL constraint might reduce in size with some more careful analysis.

Still need to choose a wing loading. For now, take $\frac{W_0}{S} = 2.2$ kPa

This allows some margin on the landing requirement of 2.3 kPa.

Initial Sizing Summary.

$P_0/W_0 = 29.4$ m/s

$W_0/S = 2.2$ kPa

$P_0 = 1.82$ MW $\qquad W_0 = 61.8$ kN $\qquad S = 28.1\,m^2$

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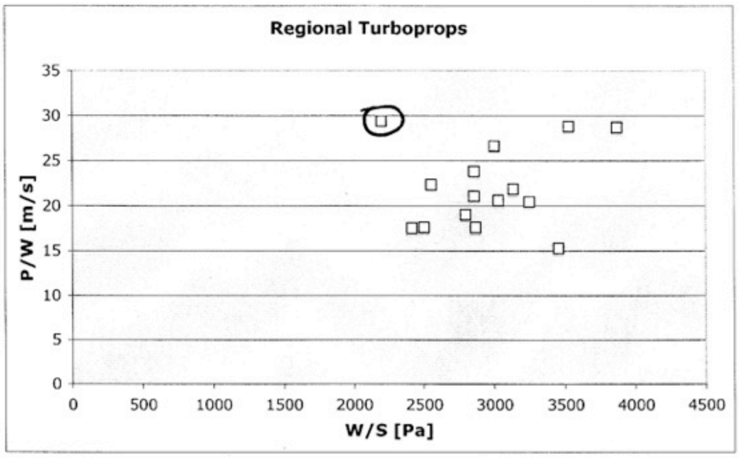
JOB TITLE

This is a suitable point to review our sizing in comparison to other available / comparable aircraft.



We see our initial sizing places our design at a lower wing loading and higher power loading than all the other designs.

This may be because our constraints are not realistic, because some of our analysis needs refinement, or because the other designs are to different performance requirements.

We'd want to do a careful check of all these things.

Another key issue is the accuracy of our weight estimation. The next stages of design involve tighter estimates of component weights.

For now, we will proceed to initial layout drawing stage.

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JOB TITLE

Preliminary Layout Details

Wing area $S = 28.1\,m^2$ Aspect ratio $A = 11$

Wing span $b = \sqrt{AS} = \sqrt{11 \times 28.1}\,m = 17.6\,m$

Weight $W_0 = 61.8\,kN$, $m_0 = 6295\,kg$.

Propeller sizing

PT6A-60A   1218 ESHP = 908 kW, 1700 rpm.

The two requirements for the propeller are that it has to be large enough to deliver the required power, while the tip speeds must not be high enough to give rise to compressibility-related losses. In practice this means keeping the total (helical path) tip speed below about 290 m/s.

Quasi-empirical size estimate (Raymer) for 4-bladed props

$\underline{D \simeq 0.49\,P^{0.25}}$ where P is in kW, D is in m

$= 0.49(908)^{0.25}\,m$
$= 2.67\,m.$

The equivalent tip speed at 140 m/s cruise : $V_{helical} = \sqrt{(\pi n D)^2 + (V_c)^2} = \sqrt{\left(\pi \times \frac{1700}{60} \times 2.67\right)^2 + 140^2}\,m/s$
$= 276\,m/s$: too high / marginal.

Try a standard size: 2.4 m ϕ. $V_{helix} = \sqrt{\left(\pi \times \frac{1700}{60} \times 2.4\right)^2 + 140^2}\,m/s = \underline{255\,m/s}$. OK.

<u>4 bladed, 2.4 m ϕ prop</u>.

Fuselage length   Supplied (Raymer) correlation for twin turboprops is

$l_f = a\,m_0^{\,c}$ where m_0 in kg, $a = 0.169$, $c = 0.51$.

$= 0.169\,(6295)^{0.51}\,m$
$= \underline{14.6\,m}$ cf $b = 17.6\,m$.

Tail moment arms

L_{HT} . Supplied rule-of-thumb (Raymer): L_{HT} is 50–55% of l_f.

Say $L_{HT} = 0.525 l_f = 7.67\,m$. Assume $L_{VT} \sim L_{HT}$ too.

Tail surface areas. Supplied guidelines for twin TPs:

$$C_{HT} = \frac{L_{HT} S_{HT}}{\bar{c}_W S_W} = 0.90$$

$\bar{c}_W$ = wing MAC, assume this is close to geometric mean

$$C_{VT} = \frac{L_{VT} S_{VT}}{b S_W} = 0.08$$

$$\bar{c} = \frac{S}{b} = \frac{28.1}{17.6}\,m = 1.60\,m.$$

So with L_{HT}, L_{VT} supplied already,

$$S_{HT} = \frac{0.90 \times 1.60 \times 28.1}{7.67}\,m^2 = 5.28\,m^2$$

$$S_{VT} = \frac{0.08 \times 17.6 \times 28.1}{7.67}\,m^2 = 5.16\,m^2.$$

Choose to use a T-tail to put H-tail out of prop wash. Allowed 5% reduction in areas.

$$\therefore S_{HT} = 0.95 \times 5.28\,m^2 = 5.02\,m^2, \text{ say } 5m \times 1m.$$

$$S_{VT} = 0.95 \times 5.16\,m^2 = 4.90\,m^2.$$

Passenger Compartment

Aisle width ≙ 35cm 2 – abreast seating.
" height ≙ 160 cm.

Seat pitch ≙ 85 cm × 6 = 5.1 m. height 1.6m width 1.4m

Allow 2.5m for crew provision

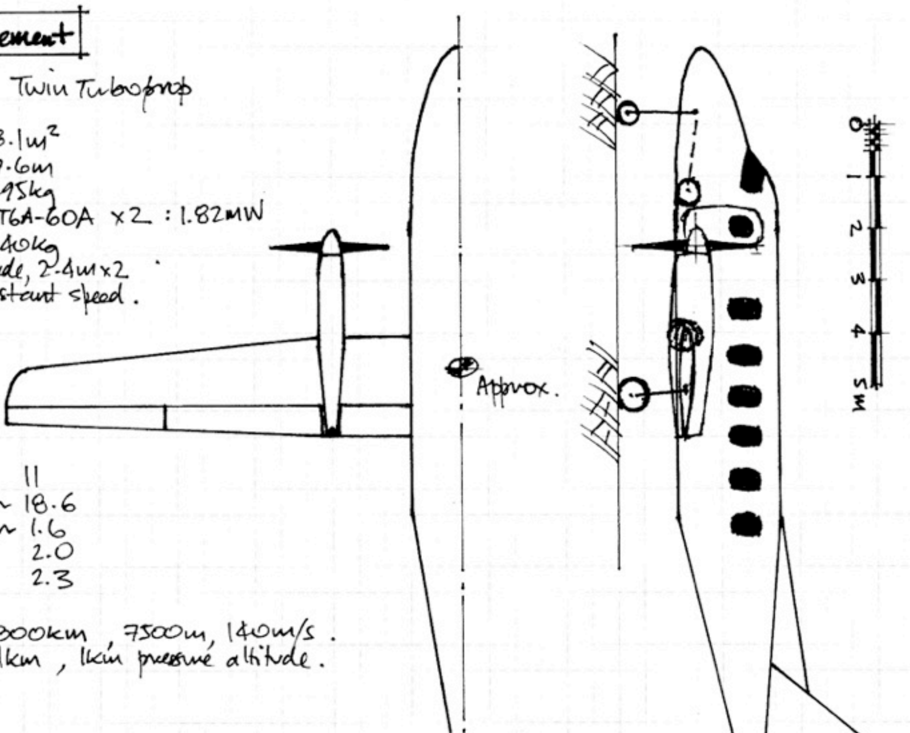
Allow 1 lavatory, 1m × 1m floor space

Compartment length: 5.1 + 2.5 + 1m = 8.6m.

General Arrangement

12 - Passenger, Twin Turboprop

Wing Area $28.1\,m^2$
Span 17.6m
MTOM 6295kg
Power 2×PT6A-60A ×2 : 1.82MW
Fuel 1240kg
Prop 4 blade, 2.4m×2 . constant speed.

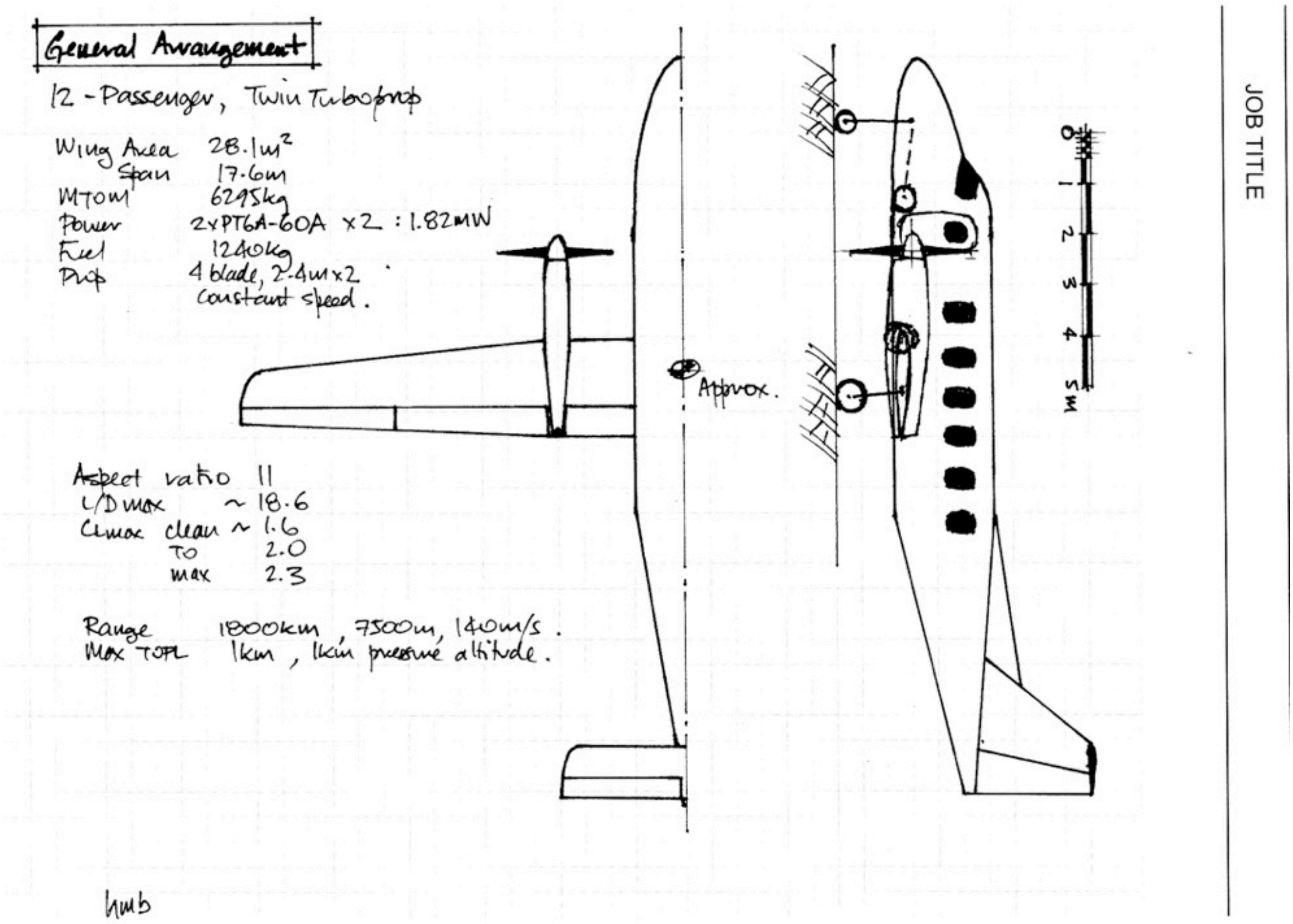


Aspect ratio 11
L/D max ~ 18.6
CLmax clean ~ 1.6
TO 2.0
max 2.3

Range 1800km, 7500m, 140m/s.
Max TOFL 1km, 1km pressure altitude.

hmb

Refined sizing using group weights – Example



Revisit our twin-turboprop 12-passenger aircraft design using group-mass sizing.

1 crew, 12 passengers. Mission analysis $\Rightarrow \frac{W_f}{W_0} = \frac{M_f}{M_0} = 0.1712$

Constraint analysis $\Rightarrow \frac{W_0}{S} = 2200$ Pa; $\frac{P_0}{W_0} = 29.4 \frac{W}{N} = 29.4$ m/s

Empty mass correlation gave $M_0 = 6295$ kg $\Rightarrow S = 28.1$ m/s

Preliminary layout gave baseline layout, fuselage shape, aspect ratio, etc.

Now
$$M_0 = \underbrace{M_{FIXED}}_{} + \underbrace{M_{VARIABLE}}_{}$$
$$= M_{FUS} + M_{PAY} + M_{OP} + M_{LIFTSUR} + M_{POWERPT} + M_{SYS} + M_{FUEL}$$

Deal with M_{FIXED} first

$\underline{M_{FUS}} = C_2\, p\,(9.75 + 5.84B)\left(\frac{2L}{B+H} - 1.5\right)(B+H)^2$ kg (pressurised fuselage)

$C_2 = 0.79$ (table 6.6)

p = cabin working differential pressure in bar : 1 bar = 100 kPa; 1 atm = 1.01325 bar.

Say (typical) cabin is pressurised to 8000 ft / 2.5 km altitude, $\delta = 0.7371$, cruise is at 7.5 km, $\delta = 0.3775$

$\therefore p = 1.01325(0.7371 - 0.3775)$ bar $= 0.3644$

From baseline layout sketch, fuselage $B = H = 1.95$ m, $L = 14.25$ m.

$$M_{FUS} = 0.79 \times 0.3644 \times (9.75 + 5.84 \times 1.95)\left(\frac{2 \times 14.25}{2 \times 1.95} - 1.5\right)(2 \times 1.95)^2 \text{ kg} = \underline{537.5 \text{ kg}}$$

$\underline{M_{PAY}}$ = passengers + baggage $= 12 \times (82+20)$ kg $= 1224$ kg (given)

$\underline{M_{OP}} = 85 n_c + F_{OP} P$ kg

n_c = number of crew (1)
P = number of PAX (12)
F_{OP} = (medium range, estimate from table), 10 kg.

$= 85 \times 1 + 10 \times 12$ kg

$= \underline{205 \text{ kg}}$

$\therefore M_{FIXED} = 537.5 + 1224 + 205$ kg $= \underline{1967 \text{ kg}}$

Now turn to variable masses

$$\underline{M_{LIFTSUR}} = C_1\left[A^{0.5} S^{1.5} \sec\Lambda_E \left(\frac{1+2\lambda}{3+3\lambda}\right)\frac{M_0}{S}\ \bar{N}^{0.3}\left(\frac{V_D}{t/c}\right)^{0.5}\right]^{0.9} \text{kg}.$$
$$= C_1\left[A^{0.5}\sec\Lambda_E\left(\frac{1+2\lambda}{3+3\lambda}\right)\bar{N}^{0.3}\left(\frac{V_D}{t/c}\right)^{0.5}\right]^{0.9}\left[\frac{g}{W_0/S}\right]^{0.45} M_0^{1.35}$$
$$= \bar{C_1}\left[\frac{g}{W_0/S}\right]^{0.45} M_0^{1.35}$$

Now C_1 = (turboprop $M_0 < 46000$ kg) $= 0.00149 - 5.8 \times P \times 10^{-6}$ kg $= 0.00142$.

$A = 11$.

$\sec\Lambda_E = 1$ (unswept wing).

$\lambda \approx 0.55$ from layout sketch (or analysis).

$\bar{N} = 1.65 \times N_{limit}$, N_{LIMIT} for civil aircraft typically $3.5 \Rightarrow \bar{N} = 5.78$

V_D = max diving speed m/s (EAS) ?? Since our maximum cruise speed @ 7.5 km was 140 m/s, Assume $V_D = 160$ m/s @ 7500 m where $\sigma = 0.4544$

V_D = EAS = TAS$\sqrt{\sigma} = 160 \times \sqrt{0.4544}$ m/s $= 108$ m/s.

For t/c, we need to choose an airfoil section. Adopt NASA GA/LS series airfoil GA(W)-1

$t/c = 0.17$

$$\therefore \bar{C}_1 = 0.00142\left[11^{0.5} \times 1 \times \left(\frac{1+2\times0.55}{3+3\times0.55}\right) \times 5.78^{0.3} \times \left(\frac{108}{0.17}\right)^{0.5}\right]^{0.9}$$

$$= 0.00142\left[3.317 \times 1 \times 0.4516 \times 1.693 \times 25.21\right]^{0.9} = 59.90\times10^{-3}$$

$$\underline{M_{LIFTSUR}} = 59.90\times10^{-3} \times \left[\frac{9.81}{2200}\right]^{0.45} \times M_0^{1.35}\ kg = \underline{5.243\times10^{-3}\ M_0^{1.35}}\ kg.$$

$$\underline{M_{POWERPT}} = C_3 M_{ENG} = C_3\left(\frac{P_0}{M_0 g}\right)\Big/\left(\frac{P_0}{M_{ENG}\,g}\right) M_0 = C_3\left(\frac{P_0}{W_0}\right)\Big/\left(\frac{P_0}{M_{ENG}\,g}\right) M_0$$

$P_0/W_0 = 29.4$ m/s from performance constraint analysis

$P_0/M_{ENG}\,g = 0.38\ \frac{kW}{N} = 380$ m/s — middle of quoted range for turboprops.

$C_3 = 2.25$ (table 6.8).

$$\underline{M_{POWERPT}} = 2.25 \times \frac{29.4}{380} M_0\ kg = \underline{0.1741\ M_0}\ kg$$

$\underline{M_{SYS}} = C_4 M_0 = \underline{0.17\ M_0\ kg}$ (estimated from table 6.9)

$\underline{M_{FUEL}} = \frac{W_f}{W_0} M_0 = \underline{0.1714\ M_0}\ kg$ (from mission analysis).

JOB TITLE

Finally $M_0 = M_{FIXED} + M_{LIFTSUR} + M_{POWERPT} + M_{SYS} + M_{FUEL}$

$= 1967 + 5.243\times10^{-3} M_0^{1.35} + 0.1741\ M_0 + 0.17 M_0 + 0.1712\ M_0$ kg

$= 1967 + 0.5153\ M_0 + 5.243\times10^{-3}\ M_0^{1.35}$ kg

Solve by fixed point iteration / successive substitution.

LHS (kg)	RHS (kg)	
6295	5915	
5915	5663	
5250	5224	
5224	5207	
5200	5191	
$\underline{5175}$	$\underline{5175}$	OK

Alternatively, rearrange and find a zero of $5.243\times10^{-3} M_0^{1.35} - 0.4847 M_0 + 1967 = 0$ kg

Also gives 5175 kg

$\underline{M_0 = 5175 kg}$

This is smaller than our original estimate of 6295 kg in the ratio $\frac{5175}{6295} = 0.822$

If the wing loading stays the same, the wing area will be reduced by the same factor. As will the power. $W/S = M_0 g/S = 2200$ Pa $\therefore \underline{S} = 5175\times9.81/2200\ m^2 = \underline{23.1}\ m^2$

($S_{new}/S_{old} = 23.1/28.1 = 0.822$ ✓).

$P_0/W_0 = P_0/M_0 g = 29.4$ m/s $\therefore \underline{P_0} = 29.4\times5175\times9.81\ W = \underline{1492\ kW}$ or 746 kW/engine.

We can easily reduce the wing area by rescaling all the wing dimensions in the ratio $\sqrt{0.822} = 0.906$.

Our original powerplant choice, PT6A-60A, has 908 kW. We could use (cheaper) $\underline{\text{PT6A-45}}$, with $\underline{875 kW}$ $\underline{202 kg}$

JOB TITLE

Since we now have a firmer estimate of powerplant mass, incorporate this into estimation.

$M_{POWERPT} = C_3 M_{ENG} = 2.25 \times 202 \times 2$ kg $= 909$ kg — treat now as part of fixed mass.

$$M_0 = M_{FIXED} + M_{POWERPT} + M_{LIFTSUR} + M_{SYS} + M_{FUEL}$$

$$= 1967 + 909 + 5.243\times10^{-3} M_0^{1.35} + 0.17 M_0 + 0.1712 M_0$$

$$= 2876 + 0.3412 M_0 + 5.243\times10^{-3} M_0^{1.35}$$

Again by successive substitution:

LHS (kg)	RHS (kg)
5175	5183
5183	5187
5190	5190

Accept $\boxed{M_0 = 5190}$ Just slightly larger than 5175 kg

SUMMARY OF CURRENT DESIGN

This is smaller than our original estimate by factor $\frac{5190}{6295} = 0.824$.

$\boxed{S} = 28.1 \times 0.824$ m^2 $= \boxed{23.2 \text{ m}^2}$ $\sqrt{0.824} = 0.908$

$\boxed{b} = 17.6 \times 0.908$ m $= \boxed{16.0 \text{ m}}$

$\boxed{\text{Power}}$: $\boxed{2\times \text{PT6A-45}}$, each with 875 kW, which exceeds requirement of

$$\frac{29.4 \times 5190 \times 9.81}{2} \text{ W} = 748 \text{ kW}$$

by a fair margin. (P_0/W_0 is now $2\times 875/(5190\times 9.81) = 34.4$ m/s > 29.4 m/s required.

For now, accept a design as originally drawn fuselage, but wing and tail surfaces reduced in size in the linear ratio 0.908.

$M_{LIFTSUR} = 5.243\times10^{-3} \times 5190^{1.35}$ kg $= 543.3$ kg This includes tail surface.

From Table 6.10, $C_S = 1.24$

$M_{WING} = \frac{M_{LIFTSUR}}{C_S} = \frac{543.3}{1.24}$ kg $= 438$ kg

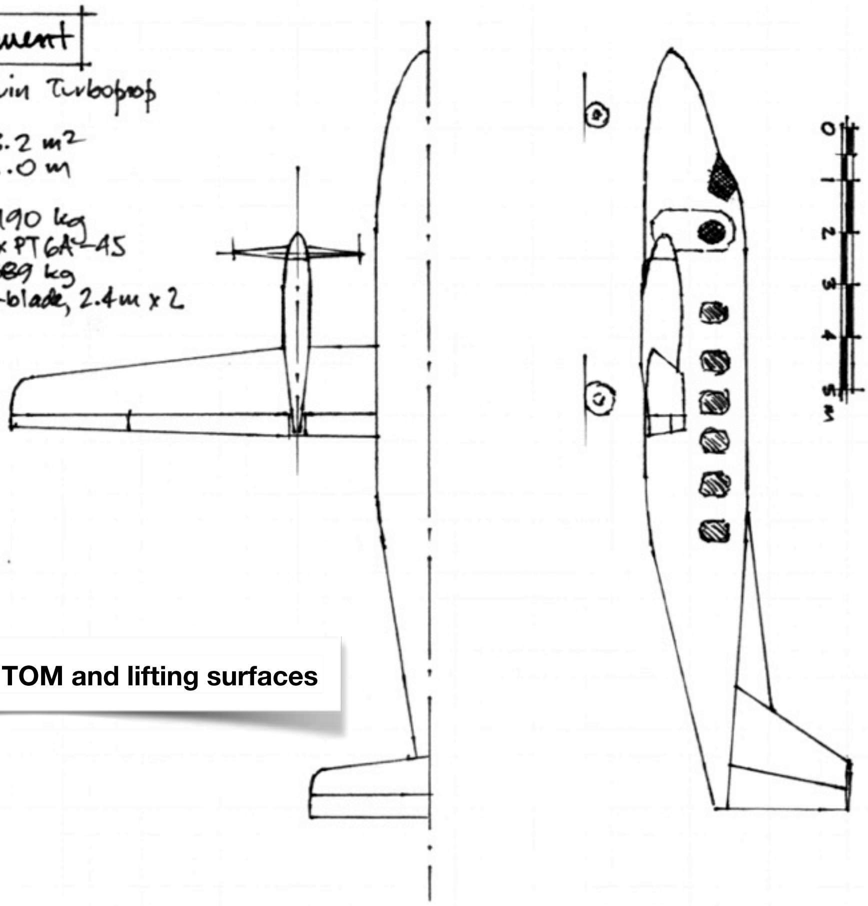
$M_{TAIL} = M_{LIFTSUR}(1 - \frac{1}{C_S}) = 105$ kg

JOB TITLE

General Arrangement

12 passenger, Twin Turboprop

Wing Area	23.2 m^2
Span	16.0 m
MTOM	5190 kg
Power	2×PT6A-45
Fuel	889 kg
Prop	4-blade, 2.4 m × 2



Revised: smaller MTOM and lifting surfaces

JOB TITLE

29

Optimisation and refined sizing – using fully-coupled mission and constraint analysis – Example

3

COUPLING CONSTRAINT ANALYSIS TO AIRCRAFT GROUP WEIGHT ESTIMATES.
— TWIN TURBOPROP REVISITED

JOB TITLE

$$M_0 = M_{FIXED} + M_{LIFTSUR} + M_{POWERPT} + M_{SYS} + M_{FUEL}.$$

Previously, we estimated this at a chosen point on the constraint diagram, where $W_0/S = 2200$ Pa, $P_0/W_0 = 29.4$ m/s – and we had left $W_f/W_0 = 0.1712$, based on best-case scenario for fuel use. To aid in choosing a design point on the constraint plot, we will now plot contours of M_0 over our constraint plot.

To do this we need to return W_0/S & P_0/W_0 into the mass estimates – and assume rubberised engine sizing.

Previously we found

$$M_0 = \underset{\text{fixed}}{1967} + \underset{\text{lifting}}{5.243\times10^{-3} M_0^{1.35}} + \underset{\text{power}}{0.1741\, M_0} + \underset{\text{system}}{0.17\, M_0} + \underset{\text{fuel}}{0.1712\, M_0}. \text{ kg}$$

$\frac{W_{fuel}}{W_0}$

which is reworked to provide

$$M_0 = 1967 + 59.90\times10^{-3}\left[\frac{9}{W_0/S}\right]^{0.45} M_0^{1.35} + \frac{2.25}{380}\frac{P_0}{W_0} M_0 + \underbrace{0.17\, M_0 + 0.1712\, M_0}\ \text{kg}$$

$$= 1967 + 167.4\times10^{-3}\left(W_0/S\right)^{-0.45} M_0^{1.35} + 5.921\times10^{-3}\left(\frac{P_0}{W_0}\right) M_0 + 0.3412\, M_0 \ \text{kg}$$

(for now, we will leave $\frac{W_{fuel}}{W_0} = 0.1712$ as previously.)

Rearrange

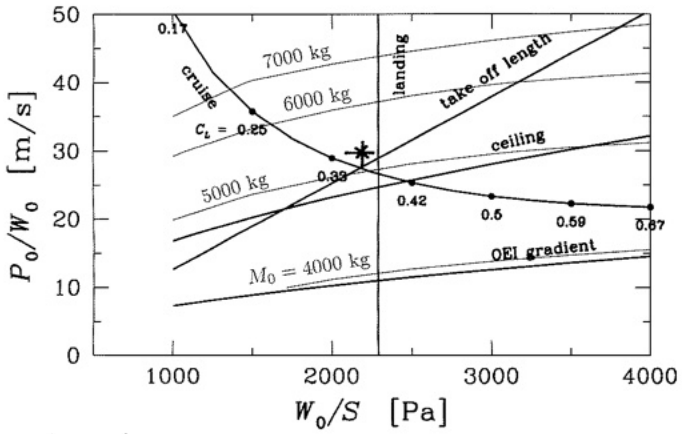
$$167.4\times10^{-3}\left(\frac{W_0}{S}\right)^{-0.45} M_0^{1.35} + 5.921\times10^{-3}\left(\frac{P_0}{W_0}\right) M_0 - 0.6588\, M_0 + 1967 = 0 \ \text{kg}$$

For each value of $\frac{W_0}{S}$ and $\frac{P_0}{W_0}$ we solve this for M_0 ... e.g. {Programmable calculator, Excel goalseek, MATLAB fzero, Python / scipy root_scalar}

M_0 (kg) for:

W_0/S (Pa) / P_0/W_0 (m/s)	1000	1500	2000	2500	3000	3500	4000
50	12180	9379	8435	7928	7605	7376	7205
40	7899	6916	6473	6211	6034	5906	5807
30	6084	5561	5301	5140	5029	4947	4883
20	5006	4679	4508	4400	4324	4266	4221
10	4275	4051	3931	3853	3798	3756	3723

And we plot contours of M_0 over our constraint diagram, as shown:



Our previous solution point ($P_0/W_0 = 29.4$ m/s, $W_0/S = 2200$) still gives $\underline{M_0 = 5190 \text{ kg}}$, as we'd expect.

With our 'cost function', M_0, at last now we have a more rational basis for choosing any particular point in the feasible region.

There is still one further problem with the analysis: we assumed $W_{fuel}/W_0 = 0.1712$, a constant, regardless of where we are on the plot. We already know that for a fixed cruise speed, changing W_0/S will change C_L, hence C_L/C_D, and so, fuel consumption.

To continue, we'll also use our more recent drag-buildup based aerodynamic estimates i.e. $C_{D0} = 0.0275$, $e = 0.86 \Rightarrow K = (\pi A e)^{-1} = 0.03365$, $C_L^* = \sqrt{\frac{C_{D0}}{K}} = 0.9040$, $\left(\frac{C_L}{C_D}\right)^* = 16.5$.

32 For our mission-based fuel use estimates, we will only alter the cruise weight fraction. W_3/W_2 (with $\prod_{0, i\neq 3}^{8} \frac{W_i}{W_{i-1}} = 0.970 \times 0.985 \times 0.995 \times 0.985 \times 0.995 \times 0.995 = 0.9271$)

and we will continue to assume that the mid-cruise weight fraction, $\beta = 0.912$.

This gives $\quad C_{L_{cruise}} = \frac{\beta}{q}\frac{W_0}{S} = 167.4 \times 10^{-6}\, W_0/S$. From C_L, get C_D, C_L/C_D

and revise $\frac{W_3}{W_2} = \exp\left(-\frac{1800 \times 10^3 \times 9.81 \times 0.085 \times 10^{-6}}{0.85\,(C_L/C_D)}\right)$ and $\frac{W_8}{W_0} = 0.9271 \times \frac{W_3}{W_2}$

finally $\frac{W_{fuel}}{W_0} = 1.06\left(1 - \frac{W_8}{W_0}\right)$.

W_0/S (Pa)	1000	1500	2000	2500	3000	3500	4000
C_L	0.1674	0.2511	0.3348	0.4185	0.5022	0.5859	0.6696
C_D	0.02844	0.02962	0.03127	0.03339	0.03599	0.03905	0.04259
C_L/C_D	5.885	8.477	10.71	12.53	13.96	15.00	15.72
W_3/W_2	0.7408	0.8120	0.8480	0.8686	0.8812	0.8889	0.8938
W_8/W_0	0.6868	0.7528	0.7862	0.8053	0.8169	0.8241	0.8286
W_{fuel}/W_0	0.3320	0.2621	0.2267	0.2064	0.1940	0.1865	0.1817

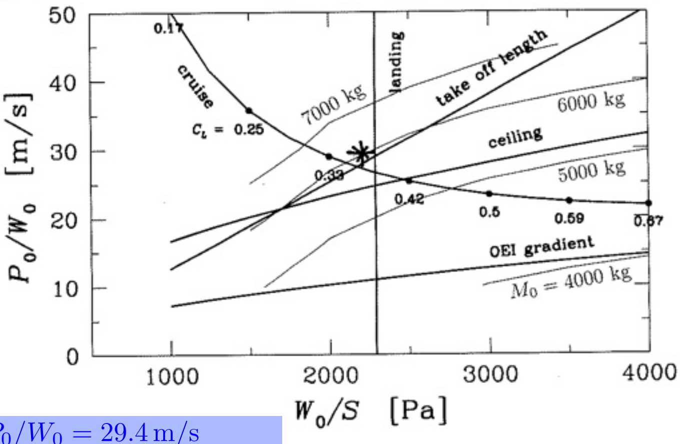
Breaking out fuel weight as a separate ratio:

$$167.4 \times 10^{-3}\left(\frac{W_0}{S}\right)^{-0.45} M_0^{1.35} + 5.921 \times 10^{-3}\left(\frac{P_0}{W_0}\right) M_0$$
$$\left(-0.8300 + \frac{W_{fuel}}{W_0}\right) M_0 + 1967 = 0 \text{ kg}$$

Again, solve for M_0:

M_0 (kg) for

P_0/W_0 / W_0/S	1000	1500	2000	2500	3000	3500	4000
50		—	12360	9597	8486	7899	7534
45		18030	9825	8148	7379	6952	6680
40		11960	8272	7118	6549	6222	6011
35		9506	7190	6338	5897	5639	5470
30	—	8014	6383	5723	5371	5162	5023
25		6978	5752	5225	4936	4762	4647
20	10400	6204	5243	4810	4569	4423	4325
15		5599	4821	4460	4255	4130	4046
10	7262	5110	4466	4159	3984	3875	3802



We see that this has a substantial effect on fuel fraction and M_0 is now very close to $\underline{M_0 = 6000 \text{ kg}}$.

$$W_0/S = 2200\,\text{Pa}, \quad P_0/W_0 = 29.4\,\text{m/s}$$
$$C_{L_{cruise}} = 167.4 \times 10^{-6} \times 2200 = 0.368 \quad (\ll C_L^*)$$

JOB TITLE

JOB TITLE

Fully-coupled analysis – reflections

1. We have now come full circle and every point on the W/S+T/W diagram includes fuel-consumption-related sizing implications because the effects of flying at different points on the drag polar (i.e. different L/D) are included.
2. Every point on the diagram now represents a distinct design within a parametric design space where we have fixed e.g. wing aspect ratio, airfoil (i.e. configuration and shape). We are only able to pick out the most rational design given those choices.
3. To make the optimisation more complete, we have to carry out parametric variations, try different configurations and design trades to see if we can do better - exactly where we will end up depends on the target of optimisation (initial purchase price, operating cost…).

Preliminary balance using group weights – Example placing CG at 0.25 MAC

Balance and CG location — 1

To balance we need both component masses and locations relative to some datum (here drawn 1m ahead of nose).

Aircraft is empty (at MWE). So include only M_{POWERPT}, M_{FUS}, M_{LIFTSUR}.

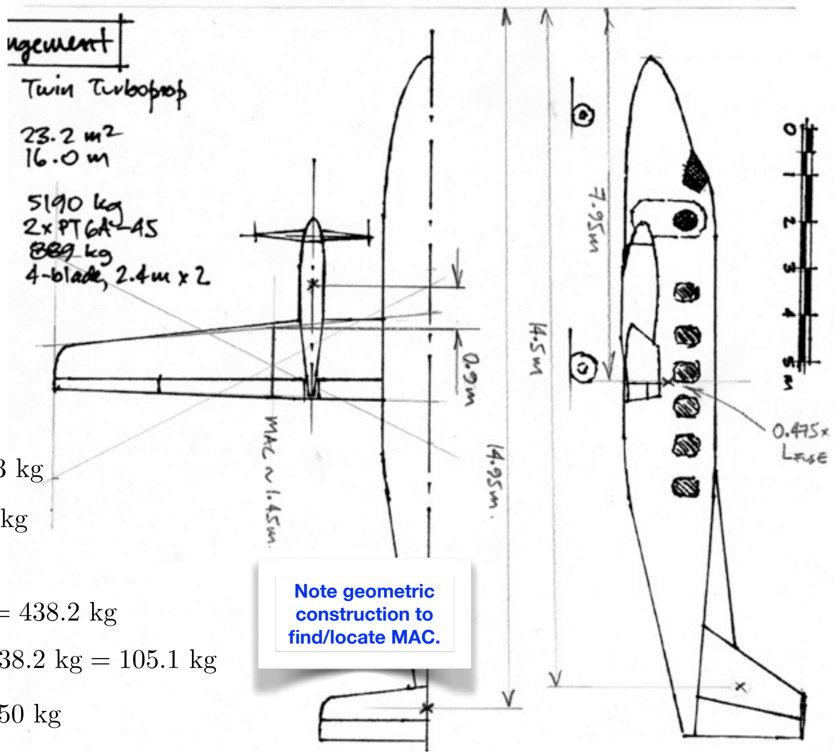
$$M_0 = 5190 \text{ kg}$$

$$M_{\text{POWERPT}} = 909 \text{ kg}$$

$$M_{\text{FUS}} = 537.5 \text{ kg}$$

$$M_{\text{LIFTSUR}} = 5.342 \times 10^{-3} M_0^{1.35} \text{ kg} = 543.3 \text{ kg}$$

$$M_{\text{MWE}} = 909 + 537.5 + 543.3 \text{ kg} = 1989.8 \text{ kg}$$



Note geometric construction to find/locate MAC.

$$M_{\text{WING}} = M_{\text{LIFTSUR}}/C_5 = 543.3/1.24 \text{ kg} = 438.2 \text{ kg}$$

$$M_{\text{TAIL}} = M_{\text{LIFTSUR}} - M_{\text{WING}} = 543.3 - 438.2 \text{ kg} = 105.1 \text{ kg}$$

Split: $M_{\text{VTAIL}} = 55.1 \text{ kg}$ $\quad M_{\text{HTAIL}} = 50 \text{ kg}$

For this type of layout we can choose to let the engines move fore and aft with the wing.

Say that the centre of M_{POWERPT} lies 0.9 m ahead of LE of MAC.

Centres of other masses are located 'by eye' and/or guidelines/calculations.

Moment equality about datum:

$$M_{\text{WING}}(X_{LE} + C_1\bar{c}) + M_{\text{POWERPT}}(X_{LE} - 0.9) + M_{\text{FUS}} \times 7.95$$
$$+ M_{\text{VTAIL}} \times 14.5 + M_{\text{HTAIL}} \times 14.95 = M_{\text{MWE}}(X_{LE} + C_2\bar{c}) \text{ kg.m}$$

where $C_1 = 0.4$, $C_2 = 0.25$, $\bar{c} = 1.45$ m

Rearrange

$$X_{LE}(M_{\text{WING}} + M_{\text{PPT}} - M_{\text{MWE}}) = M_{\text{MWE}}C_2\bar{c} - M_{\text{WING}}C_1\bar{c}$$
$$+ M_{\text{PPT}} \times 0.9 - M_{\text{FUS}} \times 9.95 - M_{\text{VTAIL}} \times 14.5 - M_{\text{HTAIL}} \times 14.95 \text{ kg.m}$$

NB: a more correct way to provide a value of C_2 is to place the CG an appropriate distance ahead of the Neutral Point. Here, for illustration purposes (and with the Neutral Point not yet known), we have used a Rule of Thumb chosen for this layout.

Solve for X_{LE}

$$X_{\text{LE}} = \frac{M_{\text{FUS}} \times 7.95 + M_{\text{VTAIL}} \times 14.5 + M_{\text{HTAIL}} \times 14.95 + M_{\text{WING}}C_1\bar{c} - M_{\text{PPT}} \times 0.9 - M_{\text{MWE}}C_2\bar{c}}{M_{\text{MWE}} - M_{\text{PPT}} - M_{\text{WING}}} \text{ m}$$

$$X_{\text{LE}} = \frac{537.5 \times 7.95 + 55.1 \times 14.5 + 50 \times 14.95 + 438.2 \times 0.4 \times 1.45 - 909 \times 0.9 - 1989.8 \times 0.25 \times 1.45}{1989.8 - 909 - 438.2} \text{ m}$$

$$X_{\text{LE}} = \frac{4534.3}{642.6} \text{ m} = 7.06 \text{ m}$$

As drawn, X_{LE} = 6.86 m.

Conclude: wing needs to go back 200 mm.

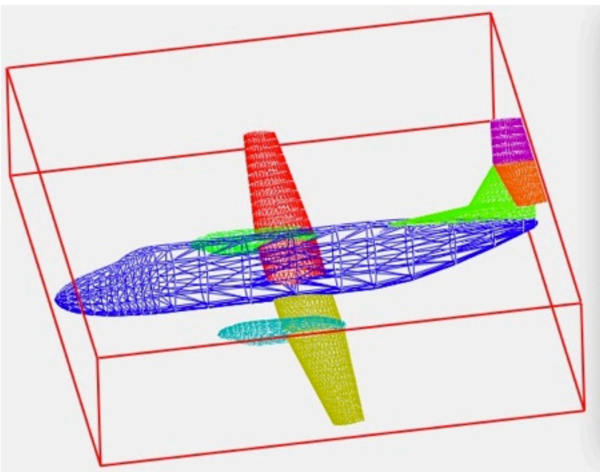
37

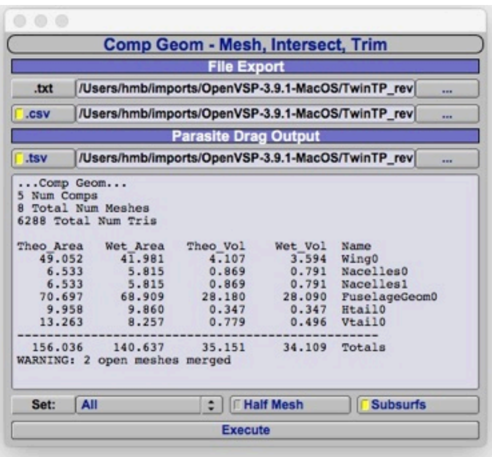
Drag polar estimation using openVSP – Example

38

Twin Turboprop: 2-term drag polar with aid of VSPaero

Construct a 2-term drag polar model for the twin turboprop already developed in class, using skin friction estimates from VSPaero for non-lifting surfaces and tails, experimental data for wing airfoil profile drag, and VSPaero's vortex lattice solver for inviscid lift-dependent drag.





Use the Comp Geom tool to get wetted areas of the major components.

Under the Wing Planform tab of the Geometry Browser, find key reference data. (But note: the chord here is NOT the MAC. That needs to be separately computed, e.g. using Mason's WingPlanAnal code. For the current geometry, MAC = 1.490 m.)

Run vspaero. Outputs relevant to drag estimation appear in the XXmodelXX_DegenGeom.history file.

Non-lifting viscous drag (taken to include tail surface drag)

Though the skin friction contributions to $C_{D,0}$ are not verified correct, we'll adopt them here to illustrate the methodology.

Skin Friction Drag Break Out:

Surface	CDo
Wing,0	0.00475
Wing,1	0.00475
Htail,0	0.00104
Htail,1	0.00104
Vtail,0	0.00196
Nacelles	0.00035
Nacelles	0.00035
Nacelles	0.00035
Nacelles	0.00035
Nacelles	0.00035
Nacelles	0.00035
Nacelles	0.00035
Nacelles	0.00035
FuselageGeom	0.00345
FuselageGeom	0.00345
FuselageGeom	0.00345
FuselageGeom	0.00345

These skin friction estimates are independent of angle of attack (and C_L).

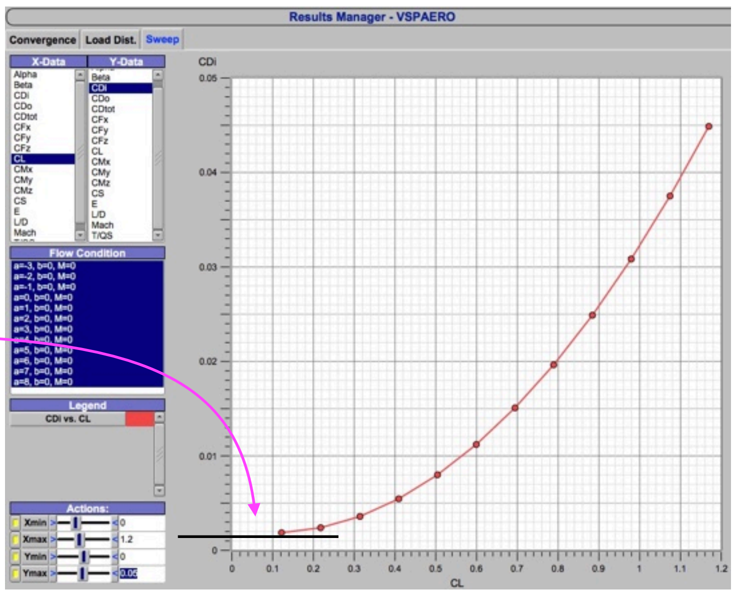
We'll use vspaero's skin friction drag estimate for everything but the wing's contribution (for which we'll adopt airfoil test data for profile drag). The total contribution is $\underline{C_{D,0,nl}}$ = 2 x 0.00104 + 0.00196 + 8 x 0.00035 + 4 x 0.00345 = $\underline{0.02064}$.

NOTE: for more recent versions of OpenVSP we could use the Parasite Drag tool instead.

Run vspaero. Outputs relevant to drag estimation appear in the XXmodelXX_DegenGeom.history file.

Inviscid induced drag

Run vspaero over a range of angles of attack that generate moderate (and positive) C_L values. Note the C_{Di} values in the .history file. Best to run in Batch mode so that outputs for all angles of attack are catenated to the .history file. The values can be plotted in the Sweep window.

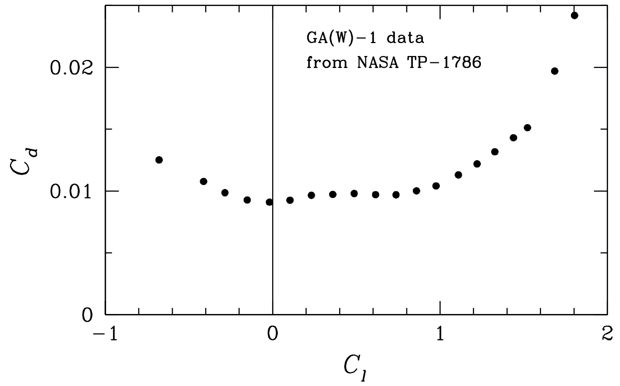


Note that C_{Di} does not asymptote to zero at C_L = 0.

That's because at overall C_L = 0,
(a) owing to the wing's washout distribution various parts of it lift either up, or down, generating induced drag, and
(b) the horizontal tail has negative lift, hence generates its own induced/trim drag.

Airfoil profile drag

This can be either generated using a 2D viscous–inviscid interaction code (e.g. XFOIL), or by adopting wind tunnel test data, if any. I've used the NASA LS GA(W)-1 airfoil series for the wing, with test data available from the NASA Technical Reports Server: NASA-TM-78709.



These are the data for the 17% thick section used at the wing root.

(The Reynolds number should be checked...)

We'll assume the polar is representative of the whole wing, even though the wing transitions to a 13% thick section at the tips.

```
...Comp Geom...
5 Num Comps
8 Total Num Meshes
6288 Total Num Tris

Theo_Area   Wet_Area   Theo_Vol    Wet_Vol  Name
   49.052     41.981      4.107      3.594  Wing0
    6.533      5.815      0.869      0.791  Nacelles0
    6.533      5.815      0.869      0.791  Nacelles1
   70.697     68.909     28.180     28.090  FuselageGeom0
    9.958      9.860      0.347      0.347  Htail0
   13.263      8.257      0.779      0.496  Vtail0
-------------------------------------------------------
  156.036    140.637     35.151     34.109  Totals
WARNING: 2 open meshes merged
```

We will only use these data for the exposed wetted area of the wing. Comparing this to its total wetted area, we see that we have to factor the profile drag contribution by approximately

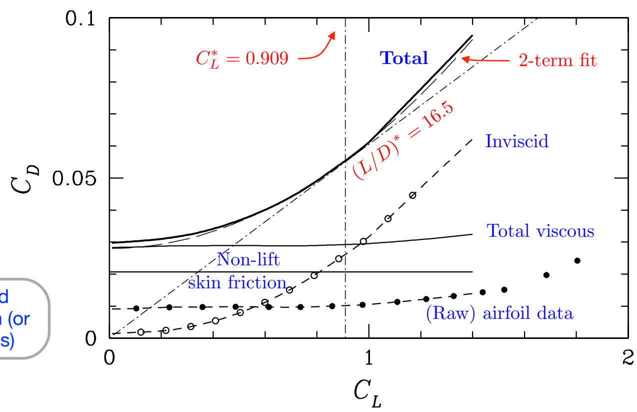
41.981/49.052 = 0.8445.

Sum up the contributions

$$
\begin{aligned}
C_D &= C_{D,\text{parasitic}} + C_{D,\text{wing}} \\
&= C_{D0,nl} + C_{D,\text{wing}} \\
&= C_{D0,nl} + C_{D,\text{profile}} + C_{D,\text{induced}} \\
&\approx C_{D0,nl} + 0.8445\, C_{d,\text{profile}}(C_L, Re_{\text{avg}}) + C_{D,\text{induced}} \\
&= C_{D0,nl} + 0.8445\, C_{d,\text{profile}}(C_L, Re_{\text{avg}}) + \frac{C_L^2}{\pi A e}
\end{aligned}
$$

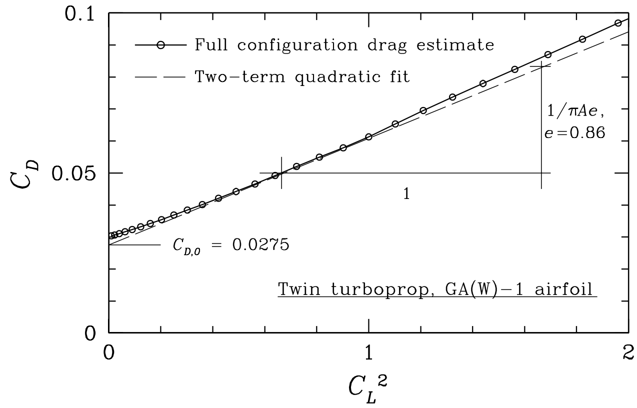
2D viscous profile drag computation (or measurement)

3D inviscid computation (or correlations)



Plot C_D vs C_L^2,

fit a 2-term quadratic over likely range of C_L.



Finally, our refined estimates:
$C_{D,0} = 0.0275$
$e = 0.86$
giving $(C_L/C_D)_{max} = 16.5$

The values initially adopted were:
$C_{D,0} = 0.02$
$e = 0.80$
and $(C_L/C_D)_{max} = 18.6$

Note that more recently, OpenVSP has introduced a Parasite Drag tool which is independent from and generally much better that the viscous drag contributions estimated by VSPaero, and which allows addition of excrescence drag contributions for unstreamlined components such as landing gear.

Thus another recommended approach is to use the Parasite Drag tool to estimate $C_{D,0}$ and VSPaero to estimate the lift-induced drag coefficient, C_{Di}, which is obtained using purely inviscid methods. From a sweep of C_{Di} vs C_L, one may obtain either K or e as a line of best fit through the origin on a C_{Di} vs $C_L{}^2$ plot.

$$K \equiv \frac{C_{D,i}}{C_L^2} \qquad\qquad e \equiv \frac{C_{D,i}\pi A}{C_L^2}$$

(That method is broadly the same as that outlined in the preceding slides: we used one tool to estimate overall viscous drag, corrected it to include an independent estimate of airfoil profile drag that included angle-of-attack contributions to viscous drag, then used VSPaero to obtain inviscid lift-induced drag, finally extracting $C_{D,0}$ and e.)

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Undercarriage sizing – Example

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Tyre and shock strut sizing – example

LANDING GEAR SIZING FOR TWIN TURBO PROP , $M_0 = 5190$ kg (11450 lb).

Already decided tricycle landing gear, main struts folding into engine nacelles. Oleo-pneumatic shock struts, aim for single wheel/strut mains, double-nosewheel.

WHEELS

For a rough indication of size, adopt Raymer's correlation for "business twin" aircraft. (Table 11.1)

$D_{cm} \simeq 8.3\, W_W^{0.251}$
$W_{cm} \simeq 3.5\, W_W^{0.216}$
where W_W is static "weight" on wheel (kg). [(Diameter) (Width)]

2 main wheels, taking total approx 90% of load so $W_W \simeq \frac{0.9 \times 5190}{2}$ kg $= 2336$ kg

$D \simeq 8.3 \times 2336^{0.251}$ cm $=$ 58.2 cm (22.9 in)
$W \simeq 3.5 \times 2336^{0.216}$ cm $=$ 18.7 cm (7.9 in)

Nose wheel dia 60-100% of mains, say 40 cm.

Probably Type VII (Extra high pressure) tyres are suitable for application. Looking in Raymer Table 11.2 a 24" × 5.5" Type VII tyre is suitable (max load 11,500 lbf : 2336 kg = 5150 lb.)

Examining Schaufele's Fig 7.7 a twin engine prop aircraft with $W_{TO} = 12000$ lb with single wheel per strut for main gear has

Not clear what "Type" { 1 × 26.6 in × 7 in (675 mm × 180 mm) Mains; 2 × 19.3 in × 6.6 in (490 mm × 170 mm) Nose wheels.

Sizes obtained by all three approaches are in reasonable agreement (and are also close to our 'guestimated' sizing on drawings made so far).

For initial layout, adopt sizes from Schaufele : Main wheels $\boxed{\text{675 mm dia} \times \text{180 mm width}}$; Nose wheels $\boxed{\text{490 mm dia} \times \text{170 mm width}}$

OLEO-PNEUMATIC STRUTS : length dictated by propeller ground clearance ; find approximate strut travel and diameter for main gear struts.

Landing loads are taken by the main gear struts, but we will take normal landing weight = 90% W_0

$$KE_{vert\,landing} = \frac{1}{2}\frac{W_{landing}}{g}V_{vert}^2 = (\eta L S)_{shock} + (\eta L S)_{tyre} = \frac{1}{2} M_{land} V_{vert}^2 = \frac{1}{2} \times 0.9 M_0 V_{vert}^2$$

$$L = N_{gear} W_{land} = 3 \times 0.9 \times 5190 \times 9.81\,\text{N} = 137.5\,\text{kN}$$

$$= \frac{1}{2} \times 0.9 \times 5190 \times 3^2\,\text{J} = 21.0\,\text{kJ}$$

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Oleo pneumatic strut, fixed orifice, $\eta_{strut} = 0.7$; Tyre $\eta_{TYRE} = 0.47$ } Raymer table 11.4.

$S_{tyre} \simeq R_{max} - R_{wheel}$, wheel diameter typically ~60% of tyre (refer tabulated data).

$$\simeq 0.5\,(D_{max} - 0.6\,D_{max}) = 0.5 \times 0.4 \times D_{max} = 0.2\,D_{max} = 0.2 \times 0.675\,\text{m} = 0.135\,\text{m}.$$

$$KE_{vert} = L\,[(\eta S)_{shock} + (\eta S)_{tyre}]$$ Rearrange to find S_{shock}.

$$S_{shock} = \frac{1}{\eta_{shock}}\left[\frac{KE_{vert}}{L} - (\eta S)_{tyre}\right] = \frac{1}{0.7}\left[\frac{21}{137.5} - 0.47 \times 0.135\right]\text{m} = 0.128\,\text{m}$$

Add safety margin of 25mm, $S_{shock} = 0.128 + 0.025\,\text{m} = 0.152\,\text{m}$, say 150mm ≈ 6"

Oleo stroke = 150mm

Oleo diameter, from Raymer.

$$D \simeq 1.3\sqrt{\frac{4\,L_{OLEO}}{P\pi}} \qquad P \simeq 12.5 \times 10^6\,\text{Pa}.$$

$$L_{OLEO} = \frac{g \times M_{LAND}}{2} \quad \text{(load spread over 2 struts)}.$$

Note: L_{oleo} does not include N.

$$= \frac{0.9 \times 9.81 \times 5190\,\text{N}}{2} = 22.9\,\text{kN}$$

$$D = 1.3 \times \sqrt{\frac{4 \times 22.9 \times 10^3}{12.5 \times 10^6 \times \pi}}\,\text{m} = 0.0627\,\text{m} \quad \text{say } 63\text{mm}$$

Oleo diameter = 63mm.