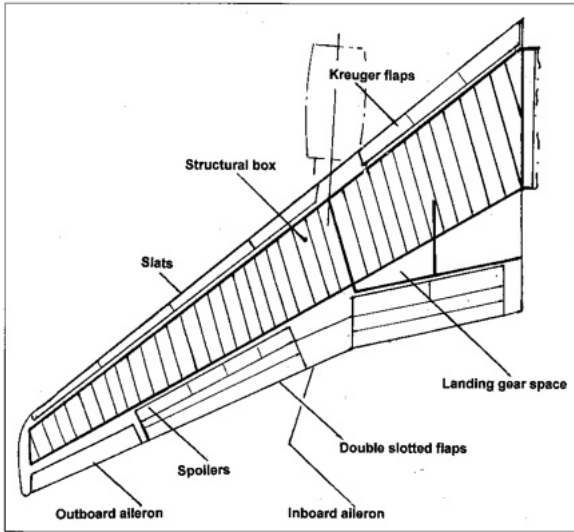


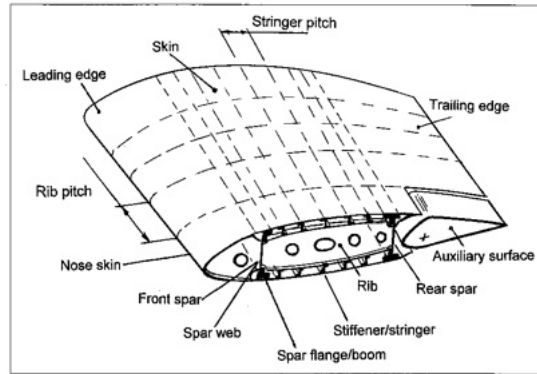
Wing structural loading and design

The three most important structural components of an aircraft; wings, fuselage and empennage are considered from the point of view of structural design as beams with variable loading along the length or span.



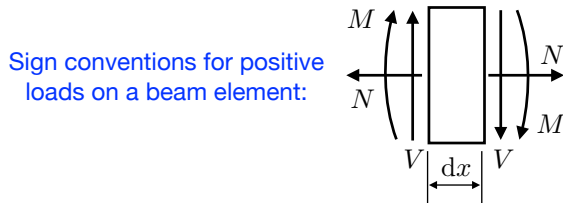
Aircraft loading & Structural layout. Howe D.

Span-wise and chord-wise beam must possess adequate bending and torsional stiffness to support loads.



1. Wing loads
2. Structural design

Wing as a simple beam – shear and bending loads



Sign conventions for positive loads on a beam element:

Integration of an impulse (point load, delta function) produces a step (Heaviside function):

$$\int_{x-\epsilon}^{x+\epsilon} \delta(x-a) dx = H(x-a) \quad (1)$$

For a distributed load (+ve downwards) $\frac{dV}{dx} = -q$ (2) where q is load/unit length.

Bending moment: $\frac{dM}{dx} = V$ (3) so that $\int_A^B dM = \int_A^B V dx$ (4)

Wing (massless) with fuselage point load W in steady level flight:

Loading

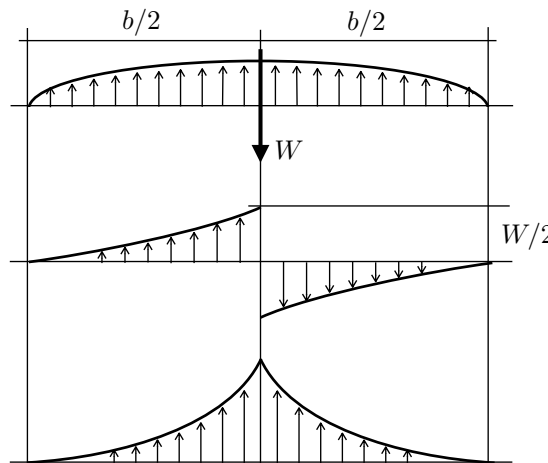
distributed lift $(-q)$
balances weight W

Shear force V

obtained by integrating (2);
for point loads, use (1)

Bending moment M

obtained by integrating (3)



$$-q = \frac{1}{2} \rho V_{\infty}^2 C_l c(x) = \rho V_{\infty} \Gamma(x)$$

$$\int_{-b/2}^{+b/2} \rho V_{\infty} \Gamma(x) dx = W \quad (5)$$

(Lift = weight)

Integrates to zero owing to (5).

Must be zero at each tip owing to (4).

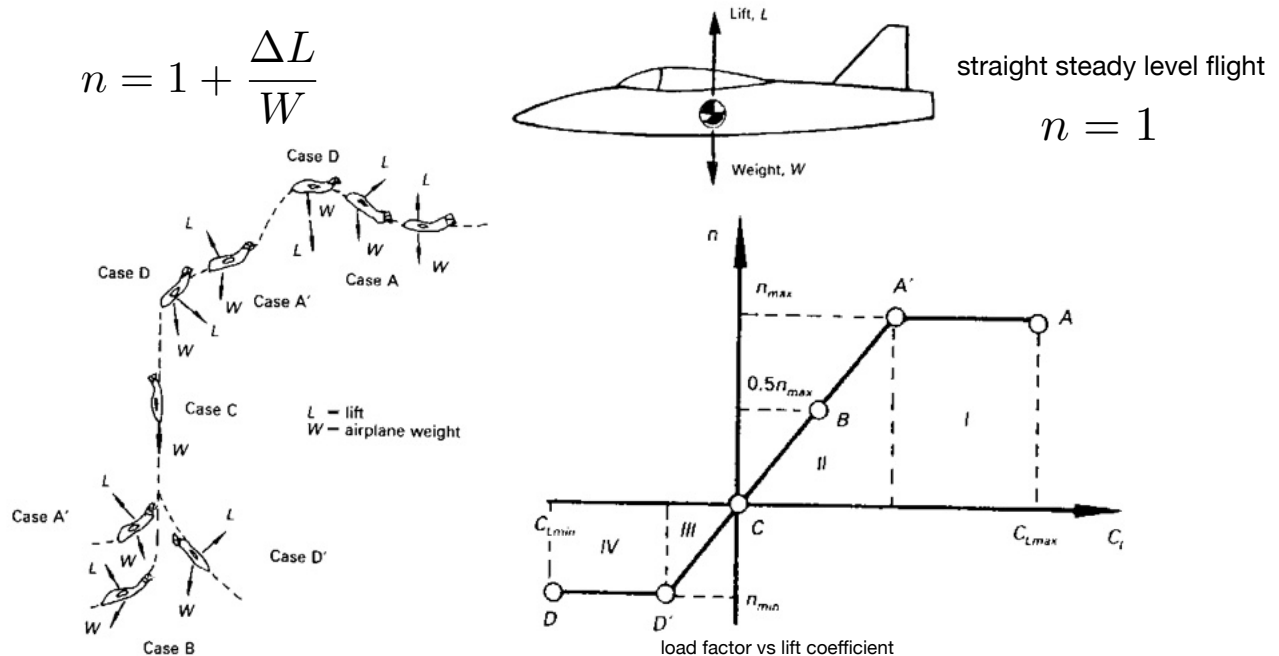
Simple to generalise for further distributed/point wing inertia loads.

Load factor

Aircraft loads are those forces applied to the airplane structural components. The determination of design loads of the wing involves both aerodynamic studies and knowledge of structural design requirements specified by the airworthiness authorities.

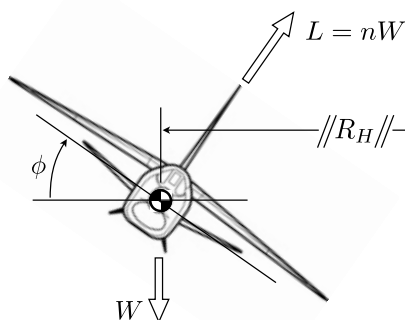
The amount of change in loads with respect straight level flight is measured in terms of the **load factor**.

In straight steady level flight, the wing lift supports the weight of the plane. However, in different maneuvers or flights through turbulence or gusts, the net load on the wing can change.



Examples of load factor

Turning performance — 1



Recall the relationships developed for turning flight:

Horizontal equilibrium $L \sin \phi = m \frac{V^2}{R_H} = mV\omega$

Vertical equilibrium $L \cos \phi = nW \cos \phi = W$

Leading to $\phi = \cos^{-1} \left(\frac{1}{n} \right)$

$$\sin \phi = \frac{\sqrt{n^2 - 1}}{n}$$

From which we obtained the following for rate and radius of turn:

$$\omega = \frac{g\sqrt{n^2 - 1}}{V}$$

$$R_H = \frac{V^2}{g\sqrt{n^2 - 1}}$$

Typically we wish to maximise the turn rate and minimise the turn radius. The first usually more important.

Now to consider the thrust requirement, we use the fundamental performance equation, simply (here):

$$T = D = \frac{1}{2} \rho V^2 S C_D = \frac{1}{2} \rho V^2 S (C_{D,0} + K C_L^2)$$

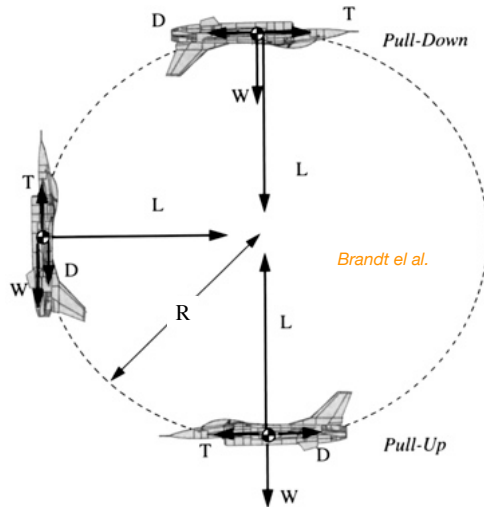
Now $L = nW = \frac{1}{2} \rho V^2 S C_L$ or $C_L = \frac{2W}{\rho S} \frac{n}{V^2}$

$$T = D = \frac{1}{2} \rho V^2 S \left[C_{D,0} + K \left(\frac{2W}{\rho S} \right)^2 \frac{n^2}{V^4} \right] = \frac{\rho}{2} S \left[C_{D,0} V^2 + K \left(\frac{2W}{\rho S} \right)^2 \frac{n^2}{V^2} \right]$$

If we set $n=1$, we recover the equation for thrust required in level flight at speed V . Increasing the load factor produces more induced drag at a given speed and hence demands more thrust.

Examples of load factor (Instantaneous) Pull-up and Pull-down

1. For instantaneous manoeuvres, we don't worry about having enough thrust to maintain airspeed. Pull-up and pull-down from level flight are typical.



$$2. \text{ At Pull-up } m \frac{V_{\infty}^2}{R} = L - W$$

$$R = \frac{mV_{\infty}^2}{L - W} = \frac{W}{g} \frac{V_{\infty}^2}{L - W} = \frac{V_{\infty}^2}{g(L/W - 1)} = \frac{V_{\infty}^2}{g(n - 1)}$$

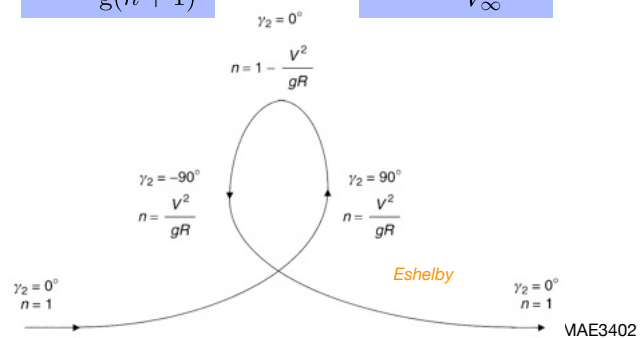
$$\omega = \frac{V_{\infty}}{R} = \frac{g(n - 1)}{V_{\infty}}$$

$$3. \text{ At Pull-down } m \frac{V_{\infty}^2}{R} = L + W$$

$$R = \frac{V_{\infty}^2}{g(n + 1)}$$

$$\omega = \frac{g(n + 1)}{V_{\infty}}$$

4. One consequence is that looping manoeuvres typically do not have a true circular flight path.



Structural design criteria

The structural criteria define the types of maneuvers, speed and loads to be considered in structural design analysis.

The criteria imposed by the airplane operator are based on conditions for which the pilot will expect the airplane to be satisfactory.

Airliners must be capable of performing well-regulated conditions in a safe manners, while military aircraft may not have well defined missions. Hence they need wider design limits.

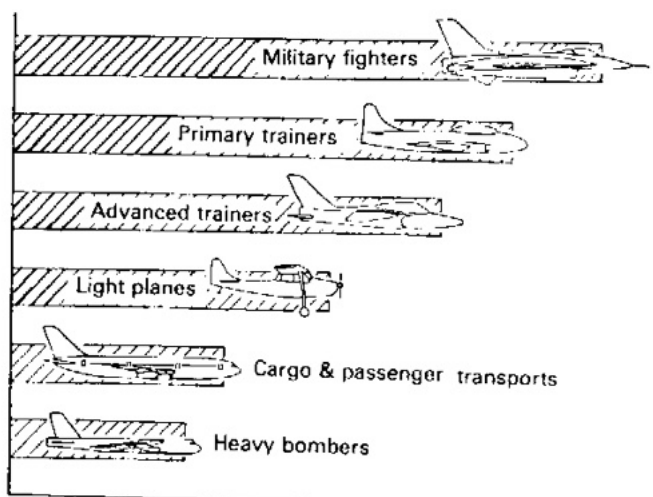


Fig. 3.1.2 Design load factor.

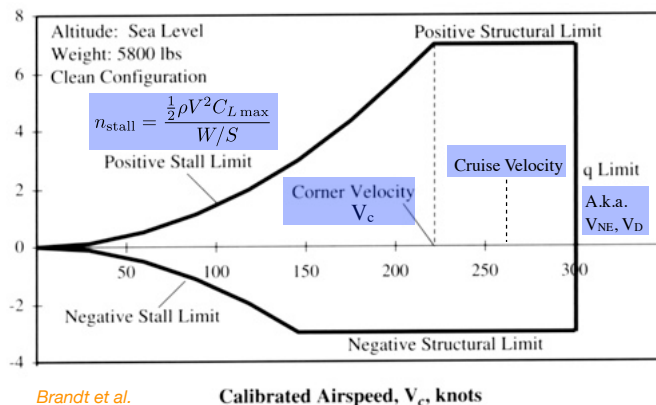


Bell X-1

$$n = \pm 18$$

The V-n diagram

1. This expresses the load-factor/speed envelope of the aircraft as determined by performance constraints (e.g. stall) and structural strength. Its limits vary with altitude and aircraft loading.
2. The load factor is derived from $L=nW$, i.e. $n=L/W$ and describes how much load the structure carries compared to the case in level flight.



3. Normal level flight has $n=1$.
4. Exceeding the structural limit n value can lead to airframe damage or breakage.
5. Exceeding the dynamic pressure (q) limit can lead to flutter or shock buffet.
6. Typically, positive structural limits are larger than the negative limits.
7. At the 'corner velocity' V_c , simultaneously at the structural strength and aerodynamic stall limits, the maximum rate of turn is achieved.

$$V_c = \sqrt{\frac{2 W}{\rho S} \frac{n_{\text{limit}}}{C_{L \max}}}$$

8. Example codified limit load factors:

McCormick

TABLE 10.1: Maximum load factors for various aircraft based on FAR-25 and 23.

Aircraft Type	Load Factor
General Aviation (normal)	$-1.25 \leq n \leq 3.1$
General Aviation (utility)	$-1.8 \leq n \leq 4.4$
General Aviation (acrobatic)	$-3.0 \leq n \leq 6.0$
Homebuilt	$-2 \leq n \leq 5$
Commercial Transport	$-1.5 \leq n \leq 3.5$
Fighter	$-4.5 \leq n \leq 7.75$

* Regulations typically require an additional structural safety factor of approx. 1.5 at the peak load factors.

TABLE 10.2: Load factors for transport aircraft based on FAR-25.

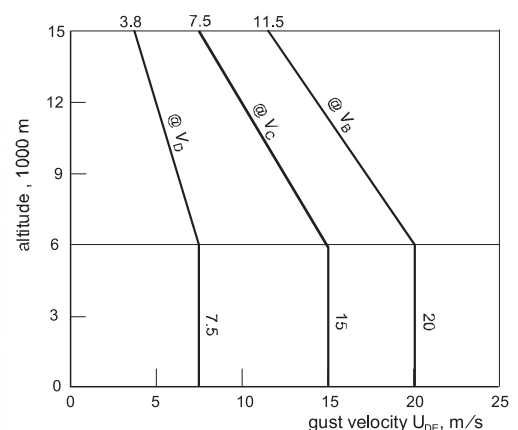
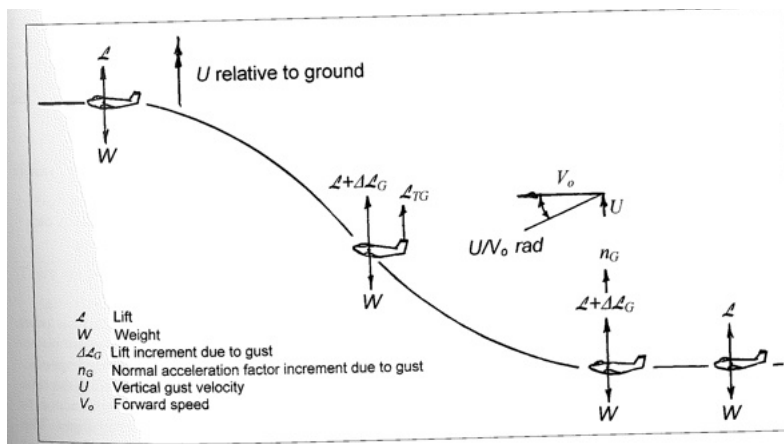
W_{TO} (lbs)	n_{\max}
≤ 4100	3.8
$4100 < W_{TO} \leq 50,000$	$2.1 + (24,000)/(W_{TO} + 10,000)$
$> 50,000$	2.5

Design Building Blocks MAE3402

Gust loadings

Gust loads are derived from the assumption that the aircraft flies through a sudden upward gust

Gust speeds are specified by structural design criteria as function of design speed and altitude



Gust velocities according to FAR 25.341

Design speed reminder

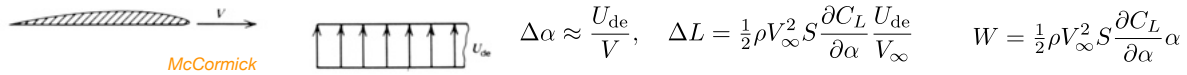
Design speed for maximum gust intensity

Design cruise speed

Design dive speed

The gust load diagram

1. Allowance is made for atmospheric turbulence in the form of gust loading factors, using gust velocities based on statistics and experience, varying with altitude.
2. Consider an aircraft encountering an idealised gust, speed U_{de} , in level flight:



$$\Delta\alpha \approx \frac{U_{de}}{V}, \quad \Delta L = \frac{1}{2}\rho V_\infty^2 S \frac{\partial C_L}{\partial \alpha} \frac{U_{de}}{V_\infty} \quad W = \frac{1}{2}\rho V_\infty^2 S \frac{\partial C_L}{\partial \alpha} \alpha$$

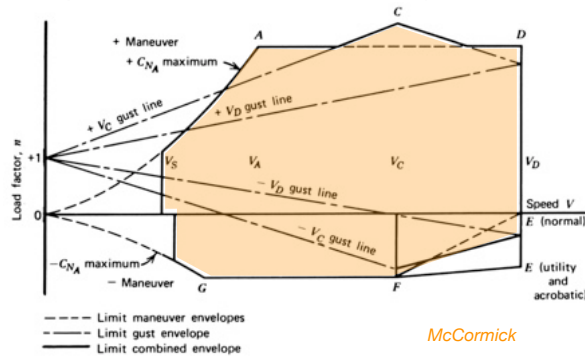
$$n = \frac{L}{W} = \frac{W + \Delta L}{W} = 1 + \frac{U_{de}}{V_\infty \alpha} = 1 + \frac{\rho V_\infty U_{de} (\partial C_L / \partial \alpha)}{2(W/S)}$$

3. A 'gust alleviation factor' K_g is applied to allow for aircraft motion/flexure in gust:

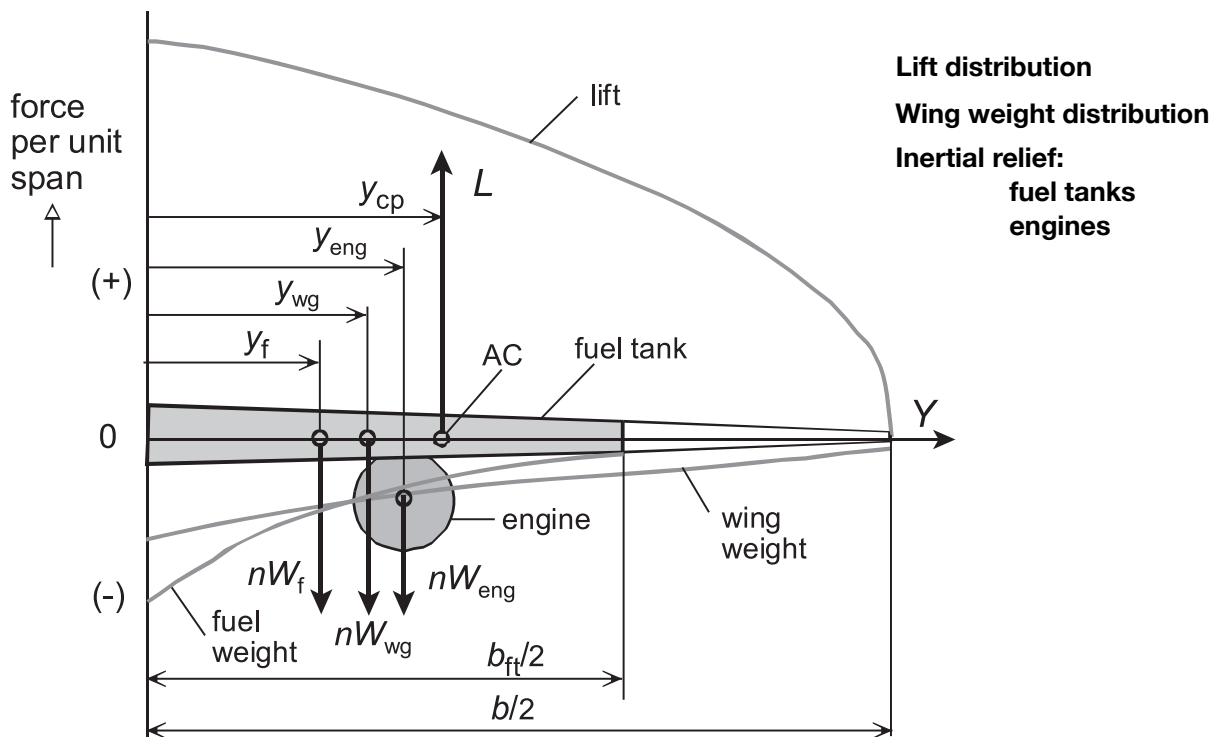
$$n = 1 + \frac{K_g \rho V_\infty U_{de} (\partial C_L / \partial \alpha)}{2(W/S)}$$

K_g is a quasi-empirical function of aircraft density relative to air density.

4. Different gust factors are applied at different flight speeds V_C , V_D .
5. Finally: another load envelope that overlays the $V-n$ envelope, and we take the worst cases.



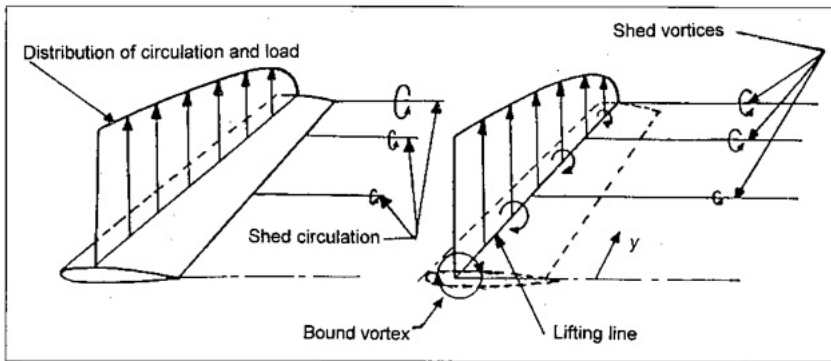
Wing loadings



Schematic of forces applied to the wing and contribution to the bending moments

Lift distribution

For unswept wings of moderate to large aspect ratio, the span wise lift distribution can be evaluated with the lifting line theory or VLM.



Aircraft loading & Structural layout. Howe D.

For wings of moderate to large aspect ratio, the span wise lift distribution can be evaluated with the lifting line theory.

$$l(y) = \rho V \Gamma(y) \quad (\text{lift per unit span})$$

Basic lift distribution

It is convenient to calculate the total loading as the sum of two separate effects
Basic loading, corresponding to zero overall lift and induced by twist of the airfoil along the span.
Additional loading, due to the lift arising from an increment in angle of attack.

$$l(y) = \rho V \Gamma(y)$$

$$l(y) = \frac{1}{2} \rho V^2 c(y) a_0 (\alpha_0 + \epsilon(y))$$

$a_0(y)$ lift curve slope of the airfoil
 $\epsilon(y)$ twist distribution of the wing
 α_0 overall zero-lift angle for the wing

For overall zero-lift

$$\int_0^{b/2} a_0(y) c(y) dy = - \int_0^{b/2} a_0(y) \epsilon(y) dy \quad (\text{additional balances basic at } C_L = 0)$$

and we define the average lift curve slope as

$$\int_0^{b/2} a_0(y) c(y) dy = S \bar{a} / 2$$

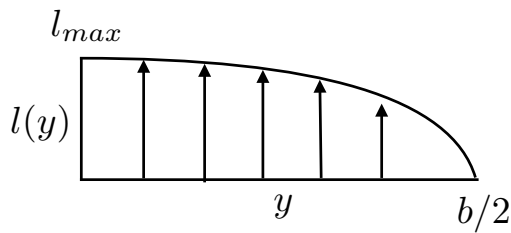
hence the overall zero-lift angle for the wing is

$$\alpha_0 = \frac{-2}{S \bar{a}} \int_0^{b/2} a_0(y) \epsilon(y) c(y) dy$$

Additional lift distribution: ideal

In lifting-line theory, the induced drag is minimized when the overall spanwise lift distribution is of semi-elliptic shape. A elliptical lift distribution can be employed a first approximation.

(l = lift per unit span)



$$(l/l_{max})^2 + (2y/b)^2 = 1$$

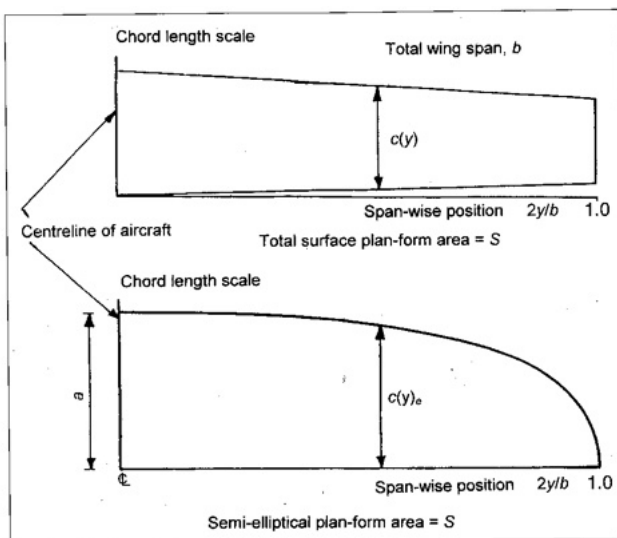
$$l(y) = l_{max} \sqrt{1 - (2y/b)^2}$$

l_{max} is usually estimated from equilibrium of forces $L = W$

$$L = 2 \int_0^{b/2} l(y) dy = \frac{1}{2} \pi l_{max} b/2 = W$$

Additional lift distribution: Schrenk's approximate method

The shape of additional distribution is the mean between the ideal semi-elliptic shape and that which would result directly from the wing planform geometry.



$$c(y)_a = (c(y)_e + c(y))$$

$$\{C_l(y)c(y)\}_a = \alpha a_0 (c(y)_e + c(y))$$

The resulting additional lift distribution consists of two parts depending on

1. Average lift curve slope and actual geometry
2. Elliptical distribution

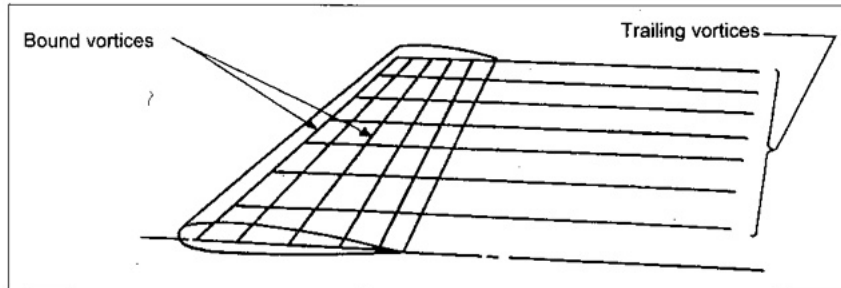
a: additional
e: elliptical

Aircraft loading & Structural layout. Howe D.
Chapter 9

Lower aspect ratio or swept wings

It is necessary to consider the distribution of the lift over the whole surface, not just the lifting line. For instance, swept wings required both chord wise and span wise direction to be analyzed simultaneously.

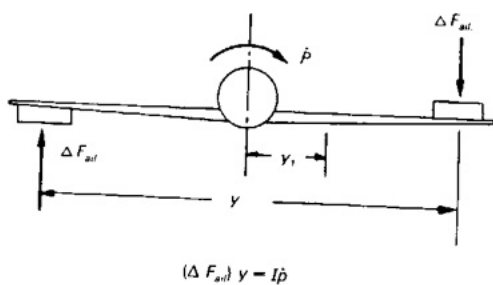
Although there is a number of pseudo-empirical methods to obtain lift-distributions of swept wings, such as the Stanton-Jones* method, vortex-lattice numerical methods are appropriated to estimate lift distributions in these cases.



* See Aircraft loading & Structural layout. Howe D. Section 9.3.2.3

Asymmetric lift distributions

Rolling maneuvers



Associated with aileron deflections.

Typical design conditions are:

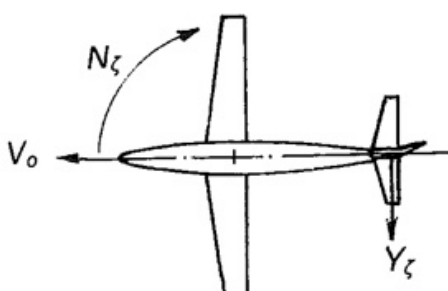
- Critical wing torsion

- Vertical tail loads by induced yaw

- Centrifugal forces on engines and fuel tanks.

Fig. 3.3.4 Forces due to deflected ailerons.

Yawing maneuvers



Associated with rudder deflection, lateral gusts or wing-mounted engine failures.

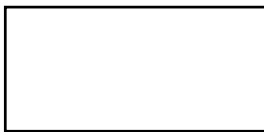
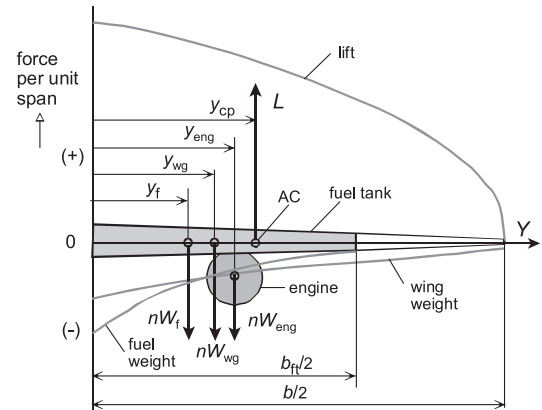
Design conditions are critical vertical tail loads

Wing weight distribution

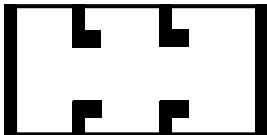
Wing weight distribution has a large influence on the structural design loads.

Wing weight distribution is a function of the chord

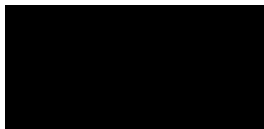
$$m(y) = Kc^\alpha(y)$$



$\alpha = 1$ Hollow wing box



$\alpha = 1.2$ The 1.2 coefficient represents that the wing structure is neither hollow nor solid. It accounts for spars and stringers



$\alpha = 2$ Solid wing box

Wing weight distribution

$$m(y) = Kc^\alpha(y)$$

The K factor is obtained from the total wing weight. The weight is usually estimated from statistics (Torenbeek) as first approx

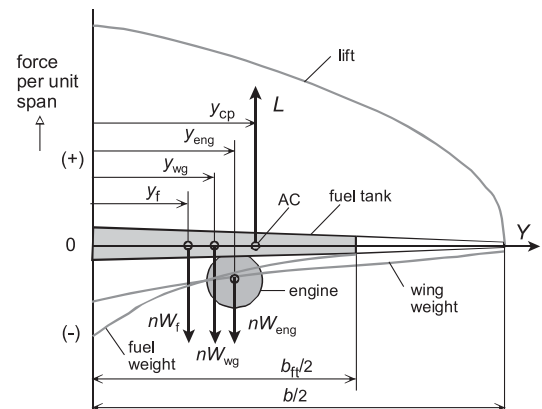
$$W_w \sim 0.115 MTOW$$

or a more precise correlation (using SI units)

$$W_w \sim 0.86b(SMZFW MTOW)^{0.25}$$

hence

$$K = \frac{W_w}{2 \int_0^{b/2} c^{1.2}(y) dy}$$



The typical designs weights are:

MTOW Maximum take-off weight
MLW Maximum landing weight
MZFW Maximum zero fuel weight
OEW Operating empty weight

Load reliefs: fuel and engines

Engines and fuel tanks act as bending reliefs. They contribute to shear force, and bending and twisting moments.

Fuel tanks are filled from wing tip to root and the fuel is consumed from wing root to tip.

Fuel tanks are installed between front and rear spars

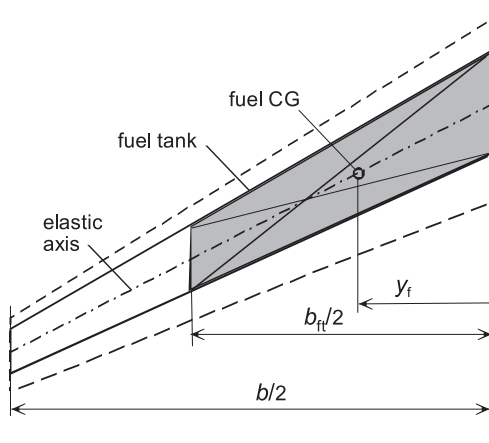
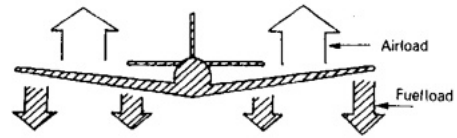
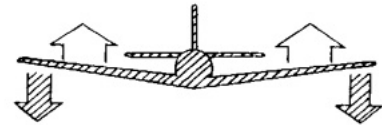


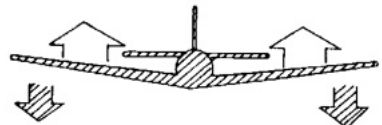
Figure 11.7 Fuel tank geometry



(a) Fuel weight provides relief to wing bending.



(b) Inboard fuel expended. Airload reduced because of reduced gross weight. Outboard fuel providing relief.



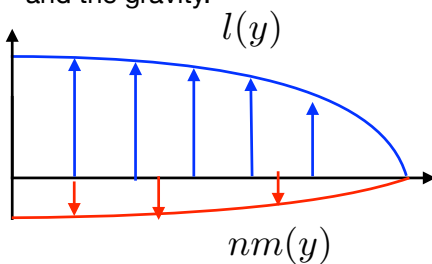
(c) Outboard fuel nearly expended. Bending relief decaying faster than airload bending; therefore, net bending increasing slightly.

Fig. 3.4.6 Illustration of the effect of fuel weight in wing.

Structural analysis of the wing

The first step consist of evaluating the contribution of the lift distribution, weight distribution, fuel weight, engine weights and additional loads to the shear force and bending and twisting moments.

The wing is considered as a beam. Notice that the masses are always multiplied by the load factor and the gravity.

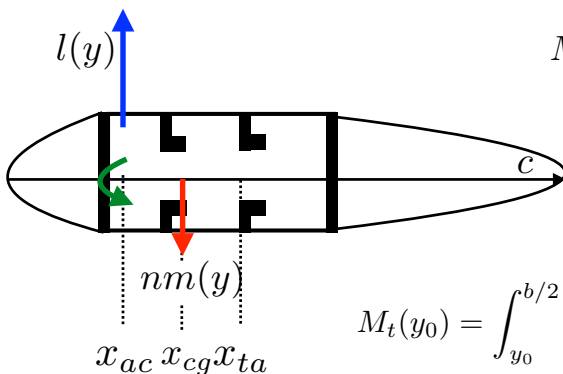


Shear force

$$q(y_0) = \int_{y_0}^{b/2} [l(y) - gnm(y)] dy$$

Bending moment

$$M_b(y_0) = \int_{y_0}^{b/2} [l(y) - gnm(y)] (y - y_0) dy$$



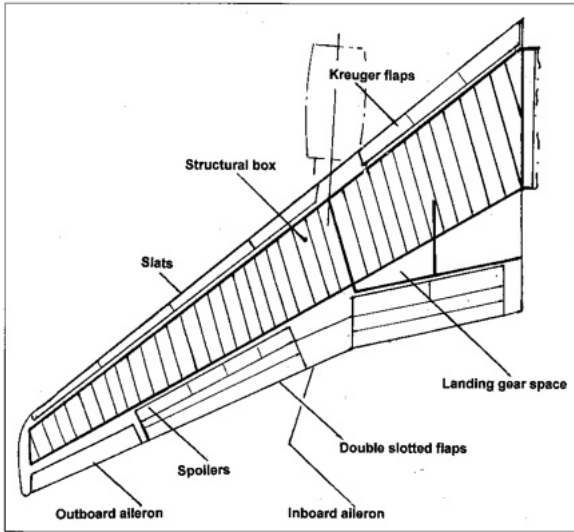
Twisting moment

$$M_t(y_0) = \int_{y_0}^{b/2} [l(y)(x_{ta} - x_{ac}) - gnm(y)(x_{ta} - x_{cg}) + \frac{1}{2}\rho V^2 c^2(y) C_m] dy$$

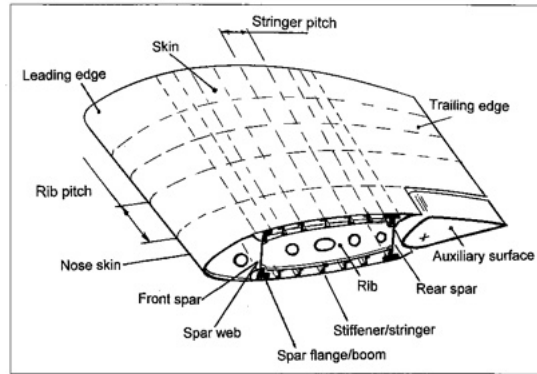
ac: aerodynamic center; cg: wing center of gravity; ta: torsional axis;

Preliminary sizing of wing components

The three most important structural components of an aircraft; wings, fuselage and empennage are considered from the point of view of structural design as beams with variable loading along the length or span.



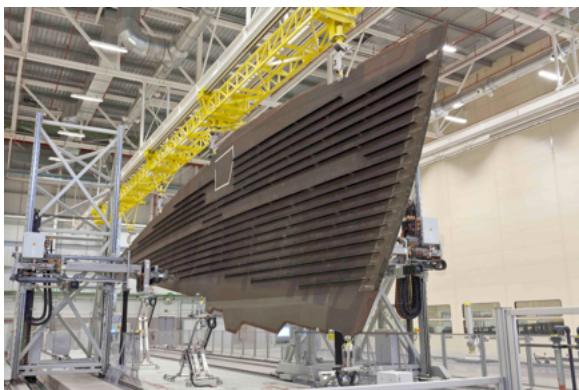
Span-wise and chord-wise beam must possess adequate bending and torsional stiffness to support loads.



Aircraft loading & Structural layout. Howe D.

1. Wing loads
2. **Structural design**

Example



Preliminary sizing of wing components

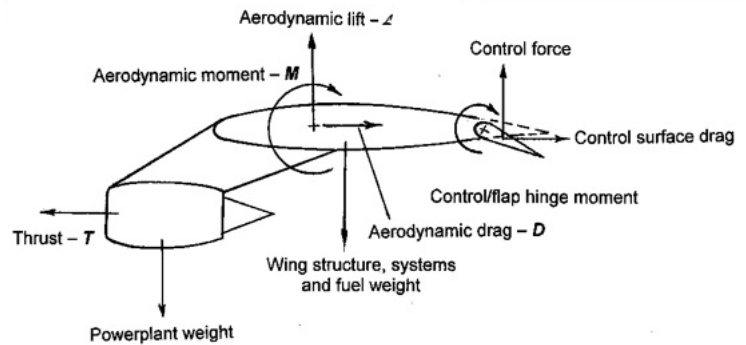
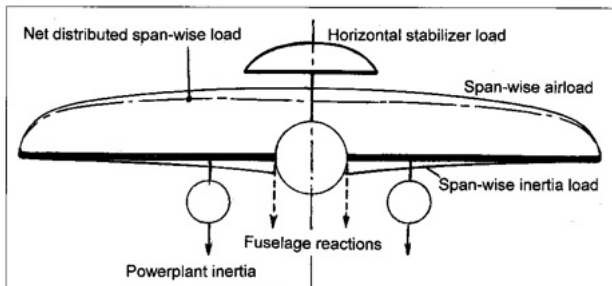
The initial sizing of structural members requires knowledge or determination of:

Loads distributions

Airframe life requirements or stiffness criteria.

An initial definition of the location of main structural members.

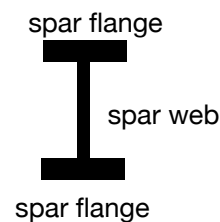
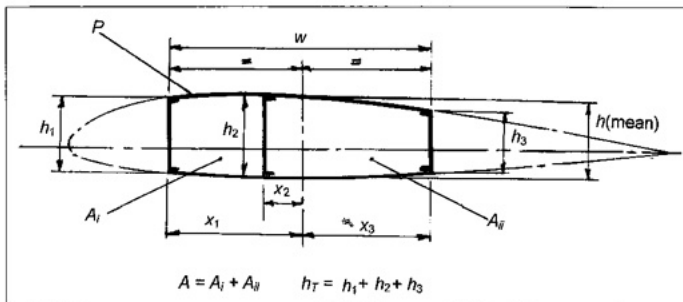
An initial choice of the main material of construction



In what follows, guidelines for preliminary sizing of wing components is based on simple spanwise estimation of loads, standard wing structure layouts and metallic or composites materials.

Cross-section of structural box

An important property of the structural cross-section is the shear center (center of twist).



The shear center depends on the size of the structural elements, hence it is not possible to determine its position until the size of the elements have been determined.

Assumptions are required to enable a prediction:

1. The cross-section is symmetrical about a horizontal plane
2. The structural box is represented by front and rear spars webs together with upper and lower skins, which reacts only to shear loads and torsion.
3. The bending moment is reacted by the spar flanges and stringers on the cover skins.

A reference position at which vertical force may be applied without causing any shear in the upper and bottom skins is obtained with these assumptions as

$$e = w / (1 + h_3 / h_1) \quad \text{Fraction of the wing box width}$$

For a rectangular wing box $e=0.5w$, which is a good approximation, the front and spars vertical web reactions are:

$$F(y) = q(y)/2$$

Torsional stiffness requirement

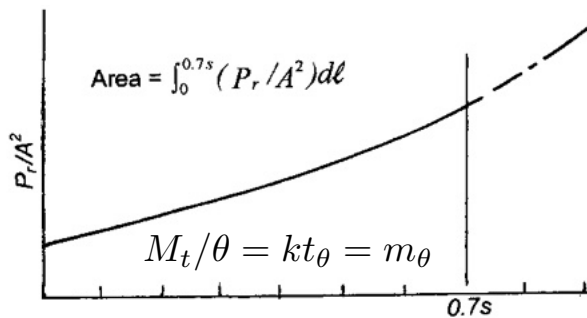
Aeroelastic requirements provided by the airworthiness authority are usually employed instead of stiffness criteria. However, these criteria are still useful for initial design phases.

Stiffness criteria is used to establish a minimum average value of the thickness of the shear material for the vertical webs and cover skins of the wing structural box.

The twist angle at each cross-section location is given by (Bred-Batho formula)

$$\theta = (M_t/t_\theta) \int \frac{P}{4GA^2} dl$$

The torsional stiffness is usually measured or defined at 0.7 of the wing halfspan. The twist angle is integrated from root to 0.7b/2



$$t_\theta = m_\theta/k$$

k comes from the integration, while the stiffness requirement is defined by design criteria.

M_t Twisting moment

A Cross-sectional area of wing box

P Perimeter of wing box

dl length of the box

t_θ mean thickness of web spars and cover skins

m_θ stiffness criteria

G shear modulus of the material

Table 13.3 Elastic moduli

Material	Tension modulus E_{xo} (MN/m ²)	Shear modulus G_{xyz} (MN/m ²)
Conventional light alloy	7.2×10^4	2.9×10^4
Aluminium-lithium alloy	7.9×10^4	3.2×10^4
Titanium	11.6×10^4	4.6×10^4
Carbon/epoxy (GFRP)		
High strength	13×10^4	3.4×10^4
High modulus	18×10^4	3.7×10^4
E glass/epoxy (GFRP)	4×10^4	0.9×10^4
Kevlar/epoxy	7.5×10^4	1.7×10^4

*Unidirectional fibres; †45°/45° weave for reinforced plastics.

Overall torsion moment

The overall torsion moment at any given cross-section is used to check the shear thickness of the spar webs and cover skins required to react the torsional loading.

The shear flow in spar webs and cover skins at each cross-section is approximately

$$Q_t = M_t/2A$$

And the mean thickness required to react the torsion moment at each cross-section:

$$t_q = T/2A\sigma_s$$

σ_s is the allowable shear stress and depends on the selection of material

Metallic materials

~50% of the ultimate tensile stress

Aluminum 2024-T6, $\sigma_s = 241 \text{ MN/m}^2$

Fiber-reinforced plastic composites (± 45 degrees angle)

- (a) Glass-fibre laminates, $\sigma_s = 60\text{--}80 \text{ MN/m}^2$.
- (b) Carbon-fibre laminate, $\sigma_s = 200 \text{ MN/m}^2$ for a quasi-isotropic lay-up to 300 MN/m^2 for all $\pm 45^\circ$ plies.

Overall bending moment

The overall bending moment at any given cross-section is used to establish the approximate value of the required material in top and bottom spar flanges or distributed flanges along the wing box.

The direct loads in top and bottom surfaces at each cross-section are given by

$$P = M_b/h$$

For a discrete boom design, where all the bending moment is reacted by spar caps, the total area of the flange on one side of the structural wing box is given by:

$$A_b = P/\sigma_b = M_b/(h\sigma_b)$$

where σ_b is the allowable stress.

For a distributed uniform flange, the required effective mean thickness is

$$t_e = M_b/(hw\sigma_b)$$

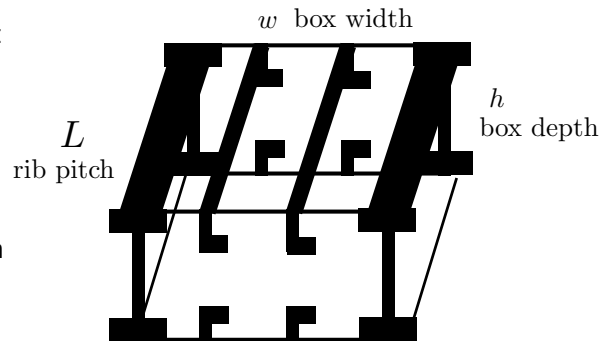
Typically, the effective thickness due to stringers is 50-100%.

Hence an common estimate of the skin thickness needed to react the bending moment is

$$t_b = 0.65t_e$$

Derivation of allowable stress needs a value for rib pitch L . An empirical optimal value is given by

$$L = 0.55(h_r)^{1/2} \quad h_r \text{ Mean depth at root chord}$$



Allowable stress

For both metallic and composites, the allowable stress depends on the load per unit width

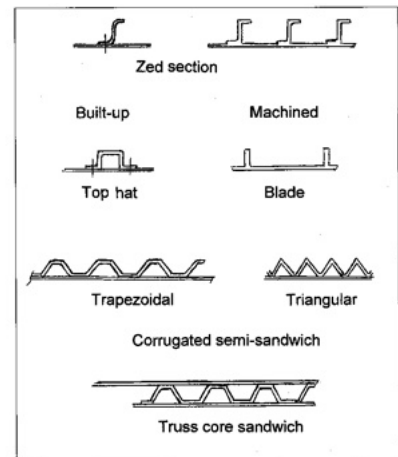
$$\sigma_b = \bar{A}F_B(P/wL)^{1/2}$$

in MN / m²

L spacing along ribs

\bar{A} function of material

F_B function of construction



Construction/material	\bar{A} MPa ^{1/2} (see also Section 13.5.4.2)			
	Plies % at:			[see Eqn. (13.2)]
	0°	±45°	90°	
Conventional light alloy with zed or integral blade stringers				138
Machined in DTD 5040 plate				180
Aluminium-lithium plate with zed stringers				200
Titanium with zed stringers (TA10, 6Al-4Va)				200
High-strength CFRP:				
Quasi-isotropic	25	50	25	150
Max. Rec. 0°	50	38	12	185
Max. Rec. ±45°	12	76	12	150
All ±45°	0	100	0	140

Note: CRFP buckling stress values allow for the additional thickness of 45° and 90° ply allowable stresses based on total laminate thickness. Carbon fibre compression strength is b moisture content and a temperature of about 45 °C.

Table 13.4 Buckling efficiency factors, F_B

Construction (see Fig. 13.4)	F_B
Zed stringer	
Built-up	0.96
Machined	1.02
Blade stringer	0.81
Top hat stringer	0.96
Trapezoidal corrugated, semi-sandwich	0.83
Triangular corrugated, semi-sandwich	0.85
Truss core sandwich	0.78

Thickness of upper and lower skins

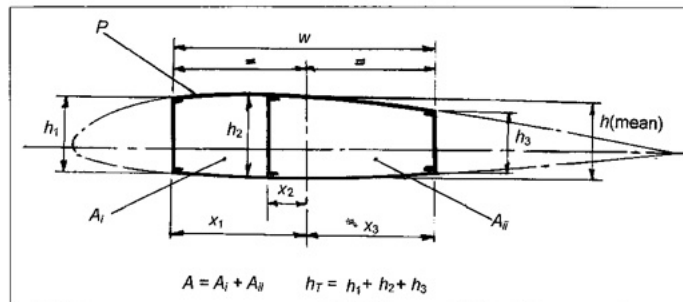
For metallic skins, the thickness of the cover skins may initially be assumed to be the greatest of that given by

- Torsional stiffness criteria
- Overall torsion moment
- Overall bending moment

In case of composite construction, it is necessary to provide sufficient directional fibres to meet the various stiffness and loading conditions

- Torsional stiffness criteria, best met ± 45 degrees angle
- Overall torsion moment, best met with ± 45 degrees angle
- Overall bending moment, best met with 0 degrees fibers.

Fibers with 90 degrees orientation are also required to react loads in the ribs.



Spar webs

The effective depth of the spars can be taken as the depth of the airfoil section at the spar positions, hence the shear flow in the webs due to the vertical shear loads is

$$Q_v = q/h_t$$

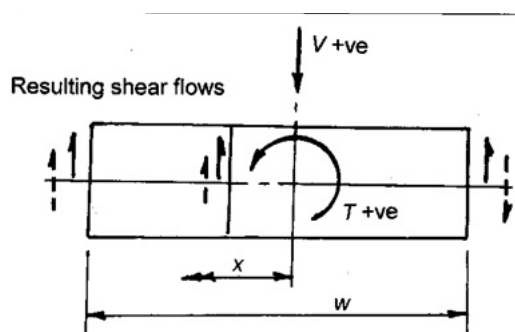
h_t is a better estimate than the depth of the rectangular wing box.

The net shear flow in the webs is then approximately given by

$$Q_w = Q_v + 2xQ_T/w$$

where x is the chord-wise location of the spar web relative to the mid-point of the box. The spar web reacts to both vertical shear loads and torsional moment. The required web thickness is:

$$t_w = Q_w/\sigma_s$$



σ_s is the allowable shear stress and depends on the selection of material

Stringer configuration

For overall bending moments, an estimation of the effective thickness that reacts due to stringers is 0.35 of the total distributed flange area. 0.65 corresponds to cover skins.

Although a more precise optimization can be carried out based on structure stability, a first estimate of the stringer configuration can be carried out with the effective thickness obtained for overall bending moments.

The most common stringers are zed or (integrated) blade section

The stringer pitch is often between 1.5 and 5 times the height of the stringer. An initial estimate is 3.5 for zed-section stringers and 2 for integrated blade stringers.

For **zed-section stringers**, the width of the flanges are around 40% of stringer height, hence the area is $1.8t_s h_s$

The assumption that the stringer area is 0.35 of the total effective area leads to

$$0.35t_e \times 3.5h_s = 1.8t_s h_s$$

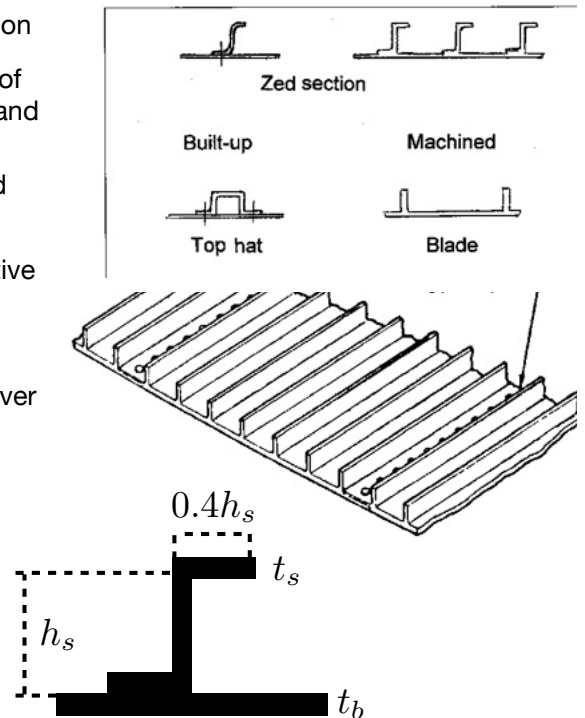
hence the thickness of the stringers is the same size as the cover skins.

$$t_s = 0.68t_e$$

The width to thickness ratio of the free flange is typically 16 to satisfy local and global buckling.

$$0.4h_s = 16t_s$$

$$h_s = 40t_s$$



Stringer configuration

The area of integrally machined **blade stringers** is simply taken as

$$h_s t_s$$

The stringer pitch in this case is usually only 2 stringer heights. This lead to a similar thickness as the cover skin

$$0.35t_e \times 2h_s = h_s t_s$$

$$t_s = 0.7t_e$$

Based on bucking considerations, the height to thickness ratio is typically 16, hence

$$h_s = 16t_s = 16t_b$$

For very thin airfoils, the effective height of the stringer may be limited by the depth of the structural wing box, and the stringers may be replaced by full depth spar webs.

